1[*Sec2.9, Linear Approximation*]

- Linearization of f at a: $L(x) = f(a) + f'(a)(x - a)$

Q1(F16): Use linearization to find a good approximation of $\sqrt{10}$ and $\sqrt{8.5}$.
Hint: consider the linearization formula for $f(x) = \sqrt{x}$ at $a = 9$.

$f'(x) = \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$, $\quad f'(9) = \frac{1}{2} \cdot \frac{1}{\sqrt{9}} = \frac{1}{6}$, $f(9) = \sqrt{9} = 3$

$L(x) = 3 + \frac{1}{6}(x - 9)$

$\sqrt{10} \approx L(10) = 3 + \frac{1}{6}(10 - 9) = \boxed{3 + \frac{1}{6}}$

$\sqrt{8.5} \approx L(8.5) = 3 + \frac{1}{6}(8.5 - 9) = \boxed{3 - \frac{1}{6} \cdot \frac{1}{2}}$

2[*Sec3.1, Extreme Values*]

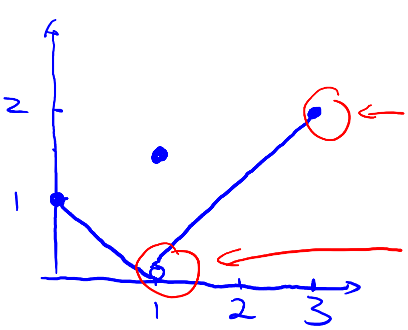
- ***Extremal Value Theorem***: If $f(x)$ is continuous on the closed, finite interval $x \in [a, b]$, then $f(x)$ possesses at least one maximum point and one minimum point.

- ***Critical points***: For a function $f(x)$, a critical point (or critical number) is a point $x = c$ where the derivative is either zero or the function is not differentiable: $f'(c) = 0$ or f' undefined

Q2.1 Find the absolute maximum and absolute minimum values of $y = f(x)$ on the interval $[0, 3]$, where

$$f(x) = \begin{cases} |x - 1|, & x \in [0, 1) \cup (1, 3], \\ 1.5, & x = 1 \end{cases}$$

(Hint: sketch the graph of $y = f(x)$)



$\leftarrow$ abs max obtained at $x=3$.

NO abs minimum.

Q2.2 Find the absolute maximum value of $f(x) = \frac{1}{2}x^2(9-2x)$ on the interval $[0, 5]$ and where the maximum is obtained.

$f(x) = \frac{1}{2}x^2 \cdot 9 - \frac{1}{2}x^2 \cdot 2x = \frac{9}{2}x^2 - x^3$, $f'(x) = \frac{9}{2} \cdot 2x - 3x^2$

$= 9x - 3x^2 = 3x(3-x) = 0$

$x = 0$ and $x = 3$.

x	0	3	5
$f(x)$	0	$\frac{1}{2} \cdot 3^2(9-6) = \boxed{\frac{1}{2} \cdot 27}$	$\frac{1}{2} 25 \cdot (9-10) = -\frac{1}{2}25$

The maximum is $\frac{27}{2}$, obtained at $\boxed{x=3}$.

Q2.3 Find the critical numbers (i.e., critical points) of the following functions

(a)

$$f(x) = \sqrt{x}$$ Domain: $[0, \infty)$

$f'(x) = \frac{1}{2} \cdot x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$ ← denominator $= 0$, $f'(x)$ D.N.E.

$\sqrt{x} = 0$, $x = 0$.

critical numbers: $\boxed{x=0}$

(b)(F16)

$$f(x) = x^{3/2} + \frac{6}{\sqrt{x}}$$ Domain: $(0, \infty)$

$= x^{\frac{3}{2}} + 6 \cdot x^{-\frac{1}{2}}$

$f'(x) = \frac{3}{2} \cdot x^{\frac{1}{2}} + 6 \cdot (-\frac{1}{2}) \cdot x^{-\frac{3}{2}}$

$= \frac{3}{2} \cdot x^{\frac{1}{2}} - 3 \cdot \frac{1}{x^{\frac{3}{2}}}$

$= \frac{\frac{3}{2} x^{\frac{1}{2}} \cdot x^{\frac{3}{2}} - 3}{x^{\frac{3}{2}}}$

$= \frac{\frac{3}{2}x^2 - 3}{x^{\frac{3}{2}}}$

critical numbers:

$\frac{3}{2}x^2 - 3 = 0$ or $x^{\frac{3}{2}} = 0$

$x = 0$

$\frac{3}{2}x^2 = 3$

$x^2 = 2$

$x = \pm\sqrt{2}$.

critical number: $\boxed{x = \sqrt{2} \text{ (only)}}$

3[*Sec3.2, Mean Value Theorem*]

- ***(MVT)*** If f is continuous on $[a, b]$ and differentiable on (a, b) then there exists $c \in (a, b)$ that satisfies $f'(c) = \frac{f(b)-f(a)}{b-a}$

Q3(F16): If the Mean Value Theorem is applied to the function $f(x) = x^2 - 2x$ on the interval $[1, 4]$, what value of c satisfies the conclusion of the theorem in this case?

$f'(x) = 2x - 2$, $f'(c) = 2c - 2 = \dfrac{f(4) - f(1)}{4 - 1} = \dfrac{4^2 - 2\cdot 4 - (1^2 - 2\cdot 1)}{4 - 1}$

$= \dfrac{16 - 8 - (-1)}{3}$

$2c - 2 = \dfrac{9}{3} = 3$

$\Rightarrow 2c = 5$

$\boxed{c = \dfrac{5}{2}}$

4.1[*Sec3.3, Derivatives and Graphs*]

- ***Increasing/Decreasing Theorem***: Let $f(x)$ be continuous on $[a, b]$.
  - If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on $[a, b]$.
  - If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on $[a, b]$.
- ***Concavity Theorem***: Let $f(x)$ be a function.
  - If $f''(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is concave up over (a, b).
  - If $f''(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is concave down over (a, b).
  - If $f''(x) = 0$ and $f''(x)$ changes its sign at $x = c$, then $f(x)$ has an inflection point at $x = c$.

4.2[*Sec3.4, Limits at Infinity*]

- ***Vertical asymptote***: $x = a$ is a V.A. of $f(x)$ if $f(x) \to \pm\infty$ as $x \to a$.
- ***Horzontal asymptote***: $y = L$ is a H.A. of $f(x)$ if $f(x) \to L$(finite) as $x \to \pm\infty$

- ***Limit at infinity***:
  - ***Limit for power functions of*** x:

$$p > 0, \quad \lim_{x\to\pm\infty} x^{\mathrm{p}} = \pm\infty \text{ (the sign depends on } p), \quad \lim_{x\to\pm\infty} x^{-\mathrm{p}} = \lim_{x\to\pm\infty} \frac{1}{x^{\mathrm{p}}} = 0$$

  - ***The highest term rule:*** Keep the highest term **in each brackets** in the numerator and denominator. Drop all the lower order terms.

4.3[*Sec3.5, Curve Sketching*]

- ***Slant asymptote:*** If a rational function $f(x) = mx + b + \frac{r(x)}{d(x)}$ via polynomial long(short) division and

$$\lim_{x\to\pm\infty} f(x) - (mx+b) = \lim_{x\to\pm\infty} \frac{r(x)}{d(x)} = 0,$$

  then $y = mx + b$ is a S.A. of $f(x)$

- ***Method for Graphing:***
  1. Determine the domain of $f(x)$. Find the x-intercepts (solve for $f(x) = 0$); and compute the y-intercept $f(0)$ if there are any(may be none).
  2. Determine the derivatives $f'(x), f''(x)$ with Derivative Rules. Find all the increasing/decreasing and concave up/down intervals. Find all local max/min and inflection points if there are any.
  3. Find all vertical/horizontal/slant asymptotes.
  4. Draw all the above features on the graph.

Q4(S16): Find all vertical and horizontal asymptotes of $f(x) = \frac{3x^2-3}{x^2+x-6}$

Horizontal: $\lim_{x\to\infty} \frac{3x^2-3}{x^2+x-6} = \lim_{x\to\infty} \frac{3\cancel{x^2}}{\cancel{x^2}} = 3.$ $\boxed{y=3}$ H.A.

Vertical: $x^2+x-6=0 \Rightarrow (x+3)(x-2)=0$

$\Rightarrow \boxed{x=-3,\ x=2.}$ V.A.

Q5(S17): Find the horizontal asymptote(s) of $f(x) = \frac{3x+\sqrt{x}}{x(1-2x)}$

$$\lim_{x\to\infty} \frac{3x+\sqrt{x}}{x(1-2x)} = \lim_{x\to\infty} \frac{3\cancel{x}}{\cancel{x}(-2x)} = \lim_{x\to\infty} \frac{3}{-2x} = \frac{3}{-\infty} = 0$$

Horizontal Asymptote: $\boxed{y=0}$

Q6(F16): Suppose

$$f(x) = \frac{x^2}{(x+2)^2}, \quad f'(x) = \frac{4x}{(x+2)^3}, \quad f''(x) = -\frac{8(x-1)}{(x+2)^4}$$

Answer the following questions or enter none in the case of no answer.

(a) Find the x and y intercepts of $y = f(x)$.

x intercept : $f(x) = 0 \Rightarrow x = 0$

y intercept: $y = f(0) = 0$

(b) Find all the asymptotes of $y = f(x)$.

$\lim_{x\to\infty} \frac{x^2}{(x+2)^2} = \lim_{x\to\infty} \frac{x^2}{x^2} = 1$.

Horizontal A: $\boxed{y=1}$

$(x+2)^2 = 0 \Rightarrow x = -2$.

Vertical A: $\boxed{x=-2}$

(c) Find all the critical points of $y = f(x)$.

$f'(x) = \frac{4x}{(x+2)^3}$ critical pts: $4x = 0$ and $(x+2)^3 = 0$

$\boxed{x = 0 \text{ and } x = -2}$

(d) Find all the interval(s) where f is increasing and where f is decreasing.

x: -3, -2, -1, 0, 1

$f'(x)$: $+$ $-$ $+$

Increasing: $(-\infty, -2) \cup [0, \infty)$

Decreasing: $(-2, 0]$

Caution: Exclude -2, since it is not in the DOMAIN.

(e) Find all the interval(s) where f is concave up and where f is concave down.

$f''(x) = 0$. $x = 1$.

x: -2, 1

f'': $+$ $+$ $-$

Concave up: $(-\infty, -2) \cup (-2, 1]$

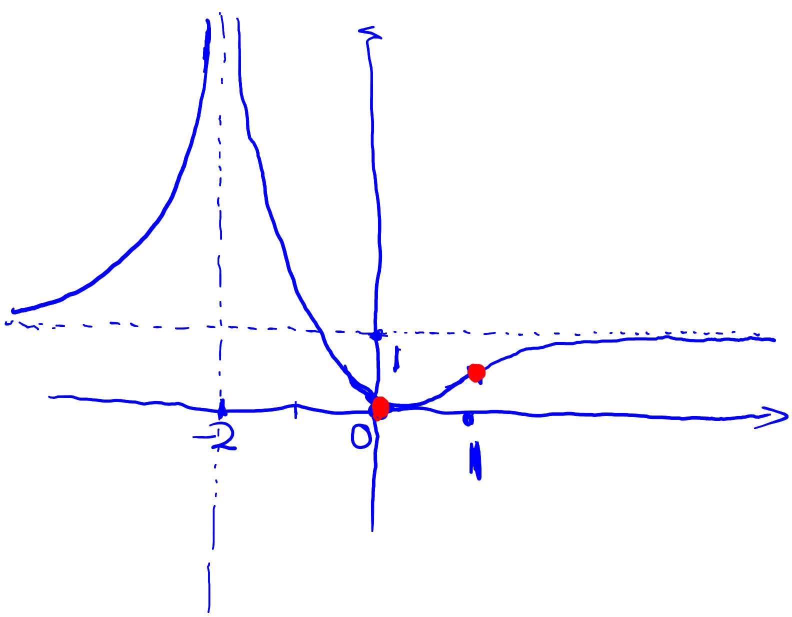
Concave down: $[1, \infty)$

(d) Find the inflection point(s) of f.

Inflection point $x = 1$.

(changing from concave up to concave down

(f) Sketch the graph of $y = f(x)$.



7[*Sec3.7, Optimization*]

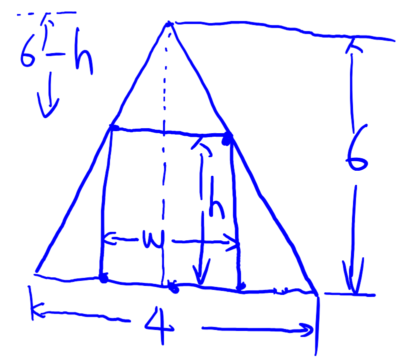
1. Draw a picture labeled with all varying quantities. Find the target function which is to be maximized or minimized. Express the target function by other quantities.

2. Write equations relating variables. Choose one as the controlling variable, and solve for all other variables in terms of it. Plug into the target function and rewrite it using only one variable. ***Determine the domain.***

3. Find the absolute maximum/minimum of the target function.

Q7 Give an isosceles triangle with base 4cm and height 6cm. A rectangle with width w and height h is inscribed with its base on the base of the isosceles triangle and its upper corners on the two legs (the two two equal sides).

(a) Express the area of the rectangle as a function of its height h.



Similar triangle: $\frac{6-h}{6} = \frac{w}{4}$.

$$w = \frac{6-h}{6} \cdot 4 = \frac{2(6-h)}{3}$$

$$A = w \cdot h = \boxed{\frac{2(6-h)}{3} \cdot h, \quad 0 \le h \le 6}$$

$$= \frac{12h - 2h^2}{3} = 4h - \frac{2}{3}h^2$$

(b) What are the dimensions of such a rectangle with the greatest possible area? And find it maximal area.

$A' = 4 - \frac{2}{3} \cdot 2h = 0, \quad 4 = \frac{4}{3}h \Rightarrow h = 3.$

$w = \frac{2(6-h)}{3} = \frac{2(6-3)}{3} = 2$

h	0	3	6
A	0	6	0

Maximal area $\boxed{A = 6}$

obtained at $\boxed{h = 3, \ w = 2.}$

Q9[*Sec3.9, Antiderivatives*]

- ***Antiderivative***. $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$. $F(x) + C$ for any constant C is called the most general antiderivative of $f(x)$
- $x^n = nx^{n-1}$, $(\sin x)' = \cos x, (\cos x)' = -\sin x, (\tan x)' = \sec^2 x, (\sec x)' = \sec x \cdot \tan x$
- ***Antiderivative Table:***

$f(x)$	$x^n, n \neq -1$	$\cos x$	$\sin x$	$\sec^2 x$	$\sec x \cdot \tan x$
Anti-D F(x)	$\frac{1}{n+1}x^{n+1}$	$\sin x$	$-\cos x$	$\tan x$	$\sec x$

- $f(x)$ is the (most general) anti-D of $f'(x)$. $f(a) = b$ can be used to determine the constant C.
- Position $s(t)$ is the anti-D of velocity $v(t)$. $v(t)$ is the anti-D of acceleration $a(t)$.

Q9.1(F16,S17): Find the most general anti-derivative of

$$y = 2\sec^2(t) + 4\cos t + 8$$

anti-D: $\boxed{2\tan t + 4\sin t + 8t + C}$

Q9.2(S16): Solve the following initial value problem: Suppose $f'(x) = \frac{1}{x^2}$ and $f(1) = 0$. Find $f(x)$.

f is the most general anti-D of $f'(x) = \frac{1}{x^2} = x^{-2}$

$\Rightarrow f(x) = \frac{1}{-2+1} \cdot x^{-2+1} + C = -1 \cdot x^{-1} + C = \frac{-1}{x} + C$

$f(1) = \frac{-1}{1} + C = 0 \Rightarrow C = 1$.

$\Rightarrow \boxed{f(x) = \frac{-1}{x} + 1}$

10[*Sec4.1, Area and Distance*]

- Approximating the area under the curve by finite rectangles; Four types of sum: Left, Right, Upper(Overestimate) and Lower(Underestimate) sums.
- Area/Integral under $y = f(x)$ on $[a, b]$ as the limit of a Riemann sum.

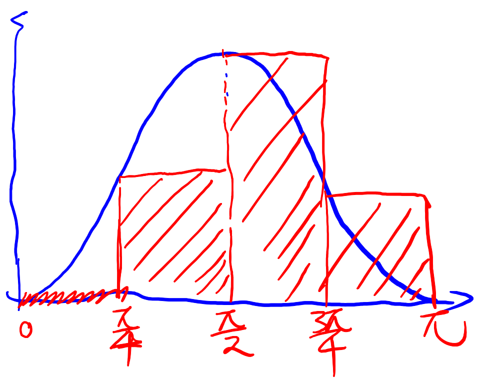
$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x,\ \ \Delta x = \frac{b-a}{n}, x_i = a + i\Delta x, i = 1, 2, \cdots, n$$

Q10.1(S15): Find (a) the Left-endpoints sum and (b) the upper sum, when we estimate the area under the graph of $f(x) = \sin x$ from $x = 0$ to $x = \pi$ using four rectangles of equal width.

$$\text{width} = \frac{\pi - 0}{4} = \frac{\pi}{4}.$$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$\sin x$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0

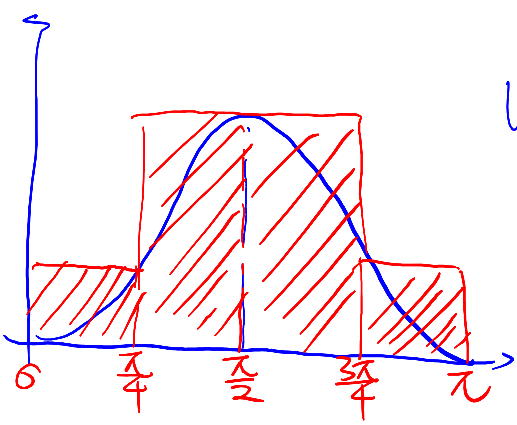
(a) left



$$\text{left-Sum} = \frac{\pi}{4}\cdot\left[f(0) + f(\tfrac{\pi}{4}) + f(\tfrac{\pi}{2}) + f(\tfrac{3\pi}{4})\right]$$

$$= \frac{\pi}{4}\cdot\left[0 + \frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2}\right] = \boxed{\frac{\pi}{4}\left[\sqrt{2}+1\right]}$$

(b) upper



$$\text{Upper Sum} = \frac{\pi}{4}\left[f(\tfrac{\pi}{4}) + f(\tfrac{\pi}{2}) + f(\tfrac{\pi}{2}) + f(\tfrac{3\pi}{4})\right]$$

$$= \frac{\pi}{4}\cdot\left[\frac{\sqrt{2}}{2} + 1 + 1 + \frac{\sqrt{2}}{2}\right]$$

$$= \boxed{\frac{\pi}{4}\left[\sqrt{2}+2\right]}.$$

Q10.2: Evaluate the sum

$$\sum_{i=0}^{2}\left(2 - 2i + \cos(\frac{\pi i}{4})\right)$$

$i=0$ $\quad$ $i=1$ $\quad$ $i=2$

$$= \left(2 - 2\cdot 0 + \cos 0\right) + \left(2 - 2\cdot 1 + \cos\frac{\pi}{4}\right) + \left(2 - 2\cdot 2 + \cos\frac{2\pi}{4}\right)$$

$$= 2 - 0 + 1 + 2 - 2 + \frac{\sqrt{2}}{2} + 2 - 4 + 0$$

$$= 3 + \frac{\sqrt{2}}{2} - 2 = \boxed{1 + \frac{\sqrt{2}}{2}}$$

11[*Sec4.2, The Definite Integral*]

- ***(Definite) Integral*** as ***Area under the curve*** and as ***the limit of a Riemann sum***

$$\int_a^b f(x)dx = \text{Area under } f(x)(\text{up to sign}) = \lim_{n\to\infty}\sum_{i=1}^{n} f\Big(a + i\frac{b-a}{n}\Big)\cdot\frac{b-a}{n}$$

- Integral Rules.

  Sum/Diff/Const.Multi.: $\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$; $\int_a^b C\cdot f(x)dx = C\cdot\int_a^b f(x)dx$
  Splitting/Fliping:

$$\int_a^c f(x)dx = \int_a^{\boxed{b}} f(x)dx + \int_{\boxed{b}}^c f(x)dx; \quad \int_a^a f(x)dx = 0; \quad \int_a^b f(x)dx = -\int_b^a f(x)dx$$

- Basic integrals from the graph:

  Rectangle: $\int_a^b 1dx = b-a$; $\int_a^b Cdx = C(b-a)$

  Half/Quater disk: $\int_{-1}^1 \sqrt{1-x^2}dx = \frac{1}{2}\pi$; $\int_{-r}^r \sqrt{r^2-x^2}dx = \frac{1}{2}\pi r^2$; $\int_0^r \sqrt{r^2-x^2}dx = \frac{1}{4}\pi r^2$

  Triangle/Trapezoid: $\int_0^b xdx = \frac{1}{2}b^2$; $\int_a^b xdx = \frac{1}{2}b^2 - \frac{1}{2}a^2$

Q11.1(F16): Which of the following definite integrals is equivalent to the following limit of a Riemann sum?

$$\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{8+\frac{5i}{n}}\cdot\frac{5}{n}$$

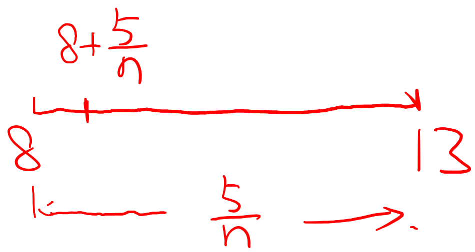
total interval length 5

A. $\cancel{\int_8^{13}\sqrt{8+x}\,dx}$; B. $\int_8^{13}\sqrt{x}\,dx$; C. $\cancel{\int_0^1\sqrt{8+5x}\,dx}$; D. $\cancel{\int_0^5 5\sqrt{8+x}\,dx}$; E. $\cancel{\int_8^{13}\sqrt[3]{8+5x}\,dx}$;

total width 1

$5\sqrt{\ }$ → $\sqrt[3]{\ }$ ↓ function

not the right function



first term should be $\sqrt{8+\frac{5}{n}}$ $f(a+\frac{5}{n})$

function: $\sqrt{x}$.

Q11.2(F16): Suppose $\int_2^5 f(x)\,dx = 3$ and $\int_2^3 f(x)\,dx = -4$. Find $\int_3^5 2f(x)\,dx$.

$$\int_3^5 = \int_3^2 + \int_2^5 = -\int_2^3 + \int_2^5$$

$$\int_3^5 2f(x)dx = \int_3^2 2f(x)dx + \int_2^5 2f(x)dx = -\int_2^3 2f(x)dx + \int_2^5 2f(x)dx$$

$$= -2\int_2^3 f(x)dx + 2\int_2^5 f(x)dx = -2(-4)+2\cdot 3$$

$$= 8+6 = \boxed{14}$$

(F15): Suppose $\int_1^4 f(x)\,dx = 5$ and $\int_2^4 f(x)\,dx = 3$. Find $\int_1^2 \Big(2f(x) - 3\Big)\,dx$.

$$\int_1^2 (2f(x)-3)\,dx$$

$$= \int_1^2 2f(x)\,dx - \int_1^2 3\,dx.$$

$$= 2\int_1^2 f(x)dx - 3\cdot(2-1).$$

$$= 2\left[\int_1^4 f(x)dx + \int_4^2 f(x)dx\right] - 3$$

$$= 2\cdot\left[\int_1^4 f(x)dx - \int_2^4 f(x)dx\right] - 3$$

$$= 2\cdot[5-3]-3$$

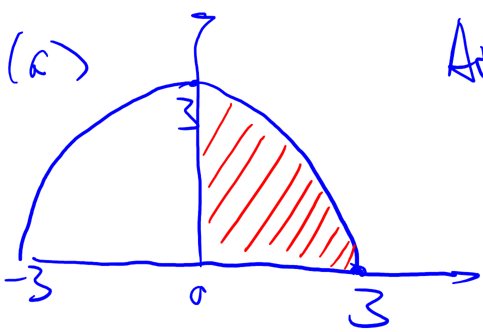
$$= 2\cdot 2 - 3$$

$$= 1$$

Q11.3(F16): Evaluate (Hint: a definite integral represents an area.)

$$\text{(a)} \int_0^3 \sqrt{9-x^2}dx, \quad \text{and (b)} \int_0^4 |3-x|dx$$

(a)



Area of the quarter disk: $\frac{1}{4}\pi\cdot 3^2 = \boxed{\frac{9}{4}\pi}$

(b)

Area of the two right triangles:

$$\frac{1}{2}\cdot 3\cdot 3 + \frac{1}{2}\cdot 1\cdot 1 = \frac{9}{2}+\frac{1}{2} = \boxed{5}$$

12[*Sec4.3, Fundamental Theorem of Calculus*]

- ***FToC P1:*** If $F(x) = \int_a^x f(t)\, dt$, then $F'(x) = \left(\int_a^x f(t)\, dt\right)' = f(x)$.
- ***FToC P1 Chain rule form:*** $\left(\int_{v(x)}^{u(x)} f(t)\, dt\right)' = f\big(u(x)\big) \cdot u'(x) - f\big(v(x)\big) \cdot v'(x)$

$$\left(\int_a^{u(x)} f(t)\, dt\right)' = f\big(u(x)\big) \cdot u'(x), \quad \left(\int_{v(x)}^{b} f(t)\, dt\right)' = -f\big(v(x)\big) \cdot v'(x)$$

- ***FToC P2:*** If $F(x)$ is an anti-D of $f(x)$, i.e., $F'(x) = f(x)$, then $\int_a^b f(x)\, dx = F(x)|_a^b = F(b) - F(a)$
- ***Antiderivative Table:***

$f(x)$	$x^n, n \neq -1$	$\cos x$	$\sin x$	$\sec^2 x$	$\sec x \cdot \tan x$
Anti-D F(x)	$\frac{1}{n+1}x^{n+1}$	$\sin x$	$-\cos x$	$\tan x$	$\sec x$

Q12.1(S17): Let

$$F(x) = \int_{\cos x}^{1} \sqrt{5 - t^2}\, dt,$$

find $F'(x)$.

$$F(x) = -\int_1^{\cos x} \sqrt{5 - t^2}\, dt$$

$$F'(x) = -\sqrt{5 - (\cos x)^2} \cdot (\cos x)'$$

$$= -\sqrt{5 - \cos^2 x} \cdot (-\sin x) = \boxed{\sqrt{5 - \cos^2 x} \cdot \sin x}$$

Q12.2(F15): Let

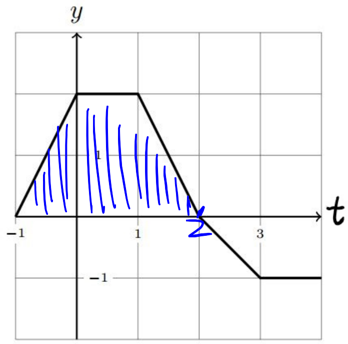
$$f(x) = \int_0^{x^2} \sqrt{1 + t^2}\, dt,$$

find $f'(x)$.

$$f'(x) = \sqrt{1 + (x^2)^2} \cdot (x^2)'$$

$$= \boxed{\sqrt{1 + x^4} \cdot 2x}$$

Q12.3(S16): The graph of a function f for $-1 \le t \le 4$ is shown below. What is the value of $\int_{-1}^{2} f(t)\,dt$.



$\int_{-1}^{2} f(t)\,dt$ = "Area" from $t=-1$ to $t=2$

$= \frac{1}{2}\cdot(1+3)\times 2 = \frac{1}{2}\times 4\times 2 = \boxed{4}$

Suppose $g(x) = \int_{-1}^{x} f(t)\,dt$. Find $g(-1), g(2)$. When does $g(x)$ attain its maximum on $[-1, 4]$?

= "Area" from $t=-1$ to $t=x$.

$g(-1) = \int_{-1}^{-1} f(t)\,dt = 0$, $g(2) = \int_{-1}^{2} f(t)\,dt = 4$.

$g(x)$ attains its maximum (maximal area) at $t=2$ since for $t \geq 2$, the "area" is negative.

• From another aspect of view, $g'(x) = f(x)$ (Fundamental Thm of calculus)

$x \le 2$, $g'(x) = f(x) \geq 0$ and $g'(2) = 0$. $x=2$ is a critical point where g attains local maximum.

$x \geq 2$, $g'(x) = f(x) \le 0$

Q12.4(F16): Evaluate

$$\int_1^2 \frac{5 - 7t^6}{t^4}\,dt$$

$$\int_1^2 \frac{5-7t^6}{t^4}\,dt = \int_1^2 \frac{5}{t^4} - \frac{7t^6}{t^4}\,dt$$

$$= \int_1^2 5\cdot t^{-4} - 7\cdot t^2\,dt$$

$$= 5\cdot\frac{1}{-4+1}\,t^{-4+1} - 7\cdot\frac{1}{2+1}\cdot t^{2+1}$$

$$= -\frac{5}{3}t^{-3} - \frac{7}{3}t^3 \Big|_1^2$$

$$\boxed{= \left(-\frac{5}{3}\cdot 2^{-3} - \frac{7}{3}2^3\right) - \left(-\frac{5}{3}\cdot 1^{-3} - \frac{7}{3}\cdot 1^3\right)}$$

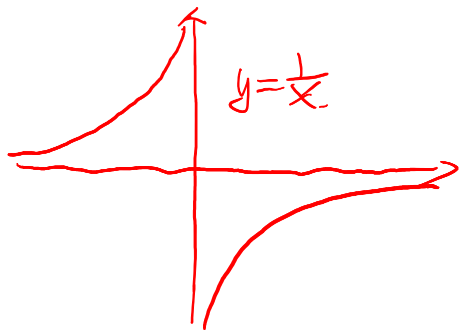
- ***Important pre-calculus facts:***

  - $$\frac{1}{x^p} = x^{-p}, \qquad x^a \cdot x^b = x^{a+b}, \qquad \frac{x^a}{x^b} = x^{a-b} = \frac{1}{x^{b-a}}$$

  - m, n are positive integers and n is even. e.g. $\sqrt[3]{x^5} = (\sqrt[3]{x})^5 = x^{\frac{5}{3}}$, $\frac{1}{\sqrt{x^3}} = \frac{1}{x^{\frac{3}{2}}} = x^{-\frac{3}{2}}$

    $x^{m/n} = (\sqrt[n]{x})^m, x \geq 0$(the domain is $[0, \infty)$); $x^{-m/n} = \frac{1}{x^{m/n}}, x > 0$(the denominator cannot be zero)

  - Graph of $y = \frac{1}{x}$



Domain $(-\infty, 0) \cup (0, \infty)$

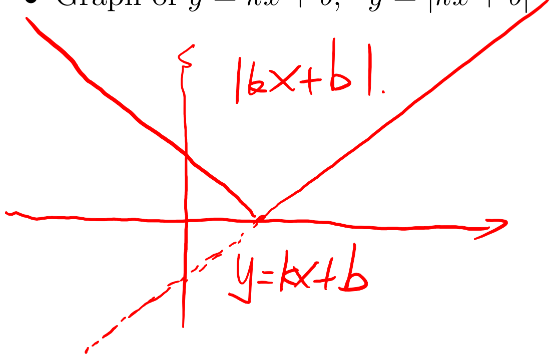
H.A: $y = 0$ $\qquad \lim_{x \to \pm\infty} \frac{1}{x} = 0$

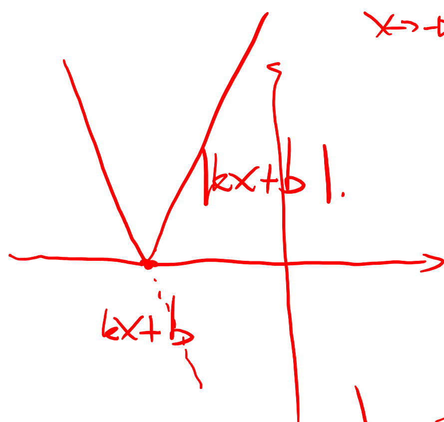
V.A: $x = 0$.

$\lim_{x \to 0^+} \frac{1}{x} = +\infty$

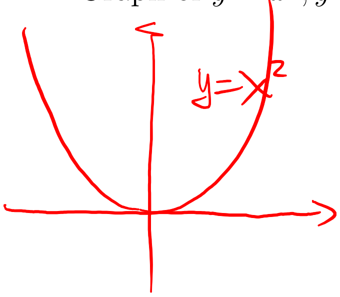
$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

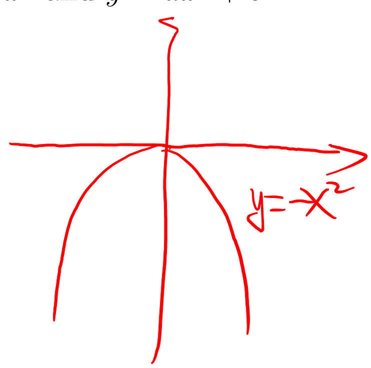
  - Graph of $y = kx + b, \quad y = |kx + b|$

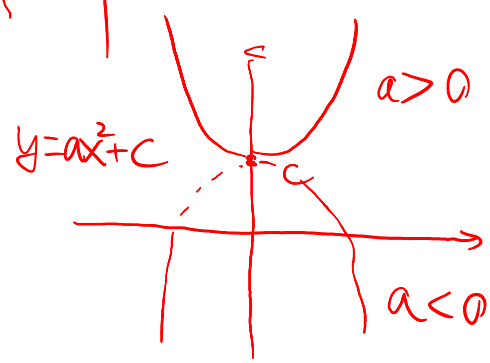




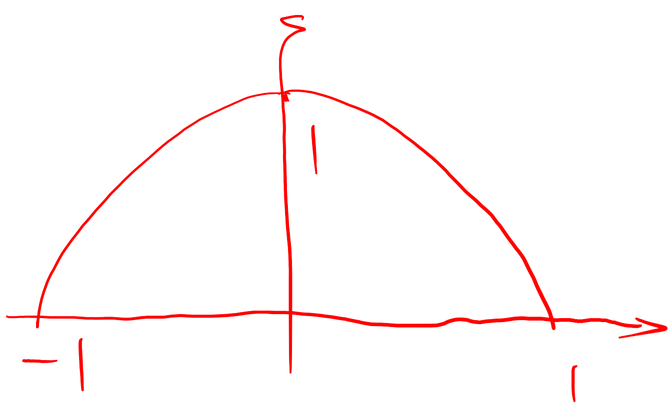
  - Graph of $y = x^2, y = -x^2$ and $y = ax^2 + c$







  - Graph of $y = \sqrt{1 - x^2}$



- $\int_a^a f(x)dx = \underline{\ 0\ }$; (flipping) : $\int_b^a f(x)dx = \underline{-\int_a^b f(x)dx}$

- splitting : $\int_a^b f(x)dx + \int_b^c f(x)dx = \underline{\int_a^c f(x)dx}$.

- $\int_a^b f(x) \pm g(x)dx = \underline{\int_a^b f(x)dx} \pm \underline{\int_a^b g(x)dx}$, $\int_a^b C \cdot f(x)dx = \underline{C\int_a^b f(x)dx}$.

- $\int_a^b 1dx = \underline{b-a}$, $\int_a^b Cdx = \underline{C(b-a)}$

- If $F(x) = \int_a^x f(t)\ dt$, then $F'(x) = f(x)$

- If $F(x) = \int_a^{u(x)} f(t)\ dt$, then $F'(x) = \underline{f(u(x)) \cdot u'(x)}$.

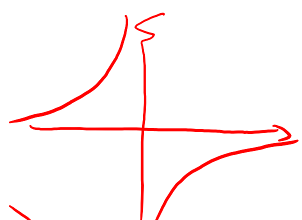
- If $F(x) = \int_{v(x)}^b f(t)\ dt$, then $F'(x) = \underline{-f(v(x)) \cdot v'(x)}$.

- If $F(x)$ is an anti-D of $f(x)$, i.e., $F'(x) = f(x)$, then $\int_a^b f(x)\ dx = \underline{F(b)-F(a)}$

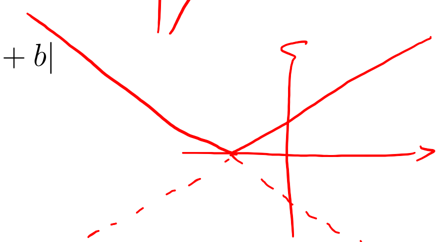
- ***Antiderivative (Integral) Table:***

$f(x)$	$x^n, n \neq -1$	$\cos x$	$\sin x$	$\sec^2 x$	$\sec x \cdot \tan x$
Anti-D F(x)	$\frac{1}{n+1} \cdot x^{n+1}$	$\sin x$	$-\cos x$	$\tan x$	$\sec x$
Definite Integral $\int_a^b f(x)dx$	$\frac{1}{n+1} b^{n+1} - \frac{1}{n+1} \cdot a^{n+1}$	$\sin b - \sin a$	$-\cos b + \cos a$	$\tan b - \tan a$	$\sec b - \sec a$

- Graph of $y = \frac{1}{x}$



- Graph of $y = |ax + b|$



- Graph of $y = x^2 - 4$, $y = -x^2 + 1$ and $y = -x^2 + x + 2$.