Practice Mid 1, Sec18

Q1[*Sec1.4, Average rate of change/Average velocity, see also Q9*] Let $f(x) = \cos x + 2$. Compute the average rate of change of $f(x)$ on the interval $[0, \frac{\pi}{2}]$

Q2[*Sec1.5/1.6, Limit and Limit Laws*] Evaluate the following limits

(a)Direct plug in-type

Suppose $\lim_{x\to 4} f(x) = 2, \lim_{x\to 4} g(x) = 3.$ Find $\lim_{x\to 4} \dfrac{xf(x)+2}{f(x) - \sqrt{g(x)}}$

(b)$\frac{1}{0}$-type/One-sided limits

$$\lim_{x\to 0^+} \frac{x-3}{x^2(x+5)}$$

(c)Absolute value

$$\lim_{x\to 1^-} \frac{|x-1|}{x-1}$$

(d)Cancellation-type

$$\lim_{x\to -2} \frac{x^2-4}{x+2}$$

Q3[*Sec1.8, Domain of continuity*] Use interval notation to indicate where $f(x)$ is continuous.

(a)

$$f(x) = \frac{x^2 - 3x + 1}{x - 3}. \qquad \text{Choose from below}$$

A. $(-\infty, +\infty)$; **B**. $(-\infty, 3) \cup (3, +\infty)$; **C**. $(-\infty, 1) \cup (1, +\infty)$; **D**. $(-\infty, 1) \cup (1, 3) \cup (3, +\infty)$.

(b)

$$f(x) = \sqrt{x + 1}. \qquad \text{Choose from below}$$

A. $(-\infty, +\infty)$; **B**. $(-\infty, -1]$; **C**. $[-1, +\infty)$; **D**. $(1, +\infty)$.

(c)

$$f(x) = \frac{(x^2 - 3x + 1)\sqrt{x + 1}}{x - 3}. \qquad \text{Use (a,b) to indicate the intervals of continuous for (c)}$$

Q4[*Sec1.8, Piecewise function*] For what value of k will $f(x)$ be continuous for all values of x ?

$$f(x) = \begin{cases} \frac{x^2 - 3k}{x - 3}, & x \le 2 \\ 8x - k, & x > 2 \end{cases}$$

A $k = 2$;

B $k = 3$;

C $k = 4$;

D $k = 5$.

Q5[*Sec1.8, Intermediate Value Theorem(IVT)*] Suppose function $h(x)$ is continuous on $[0, 4]$. Suppose $h(0) = 2, h(1) = 0, h(2) = -3, h(3) = 2, h(4) = 5$. For what value of N, the must be a $c \in (3, 4)$ such that $h(c) = N$?

A $N = 0.5$

B $N = 0$

C $N = -2$

D $N = 2.5$

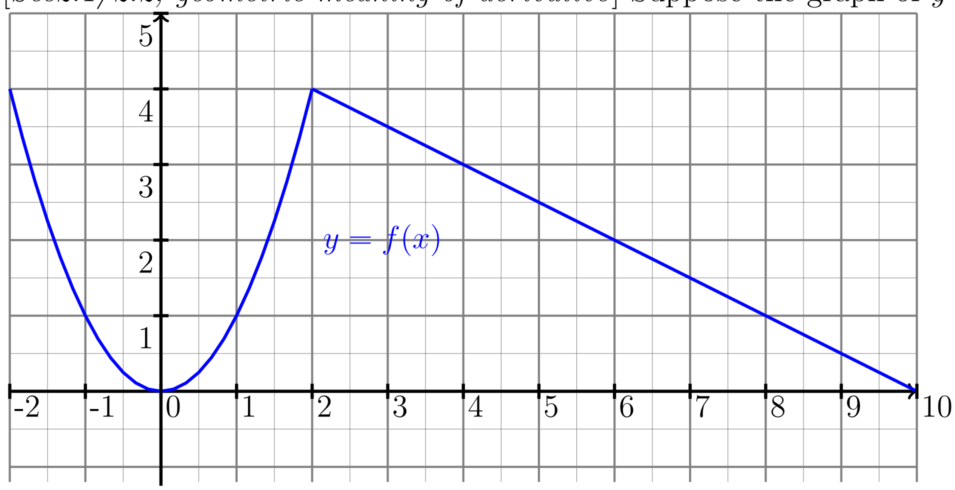
Q6[*Sec2.1/2.2, derivative at given point*] Select all true statements about the function $f(x) = \begin{cases} |x|, & x < 2 \\ 0, & x \geq 2 \end{cases}$

I $f(x)$ is differentiable at $x = 0$

II $f(x)$ is continuous at $x = 2$

III $\lim_{x \to 0} f(x)$ exists

Q7[*Sec2.1/2.2, geometric meaning of derivative*] Suppose the graph of $y = f(x)$ is given as follows:



Answer the following questions based on the above graph:

1. Find $f(0)$ and $f'(0)$. Find the equation of the tangent line of $y = f(x)$ at $(0, f(0))$.

2. Is $f(x)$ continuous at $x = 2$? Is $f(x)$ differentiable at $x = 2$? Find

$$\lim_{h \to 0^+} \frac{f(2+h) - f(2)}{h}$$

3. Find $f(6)$ and $f'(6)$. Find the equation of the tangent line of $y = f(x)$ at $(6, f(6))$.

Q8[*Sec2.1/2.2, definition of derivative*] Let $f(x) = \frac{1}{x+1}$

(a) ***[Derivative as a limit]*** Use the definition of the derivative to find $f'(x)$. (Your calculation must include computing a limit.)

(b) ***[Evaluating the derivative function at given point]*** Find $f'(2)$

(c) ***[Point-slope formula for the tangent line]*** Use part (b) to find an equation of a tangent line of $f(x)$ at $x = 2$.

Q8*[*Sec2.1/2.2, definition of derivative*] Use the definition of the derivative to find $g'(1)$ for $g(x) = 2\sqrt{x}$.

Q9[*Sec2.3/2.4/2.5, Differentiation Formulas/Laws*] Find the derivatives of the following functions. Do not need to simplify.

(a)[Linear Rule+Power functions]

$$T(x) = 2\sqrt{x} - \frac{1}{2\sqrt{x}}$$

(b)[Product Rule+Power functions]

$$g(t) = (-1 + 2t)(\sin t + 2)$$

(c)[Trig functions+Chain Rule]

$y = \sin(x^2)$

(c*)[Trig functions+Chain Rule]

$y = \sin^2(x)$

(d)[Quotient Rule+Trig functions+Chain Rule]

$$f(t) = \frac{3t}{\tan(t^2 - 1)}$$

(e)[Trig functions+Double Chain Rule]

$f(x) = 3\sec(\cos(1 - 2x))$

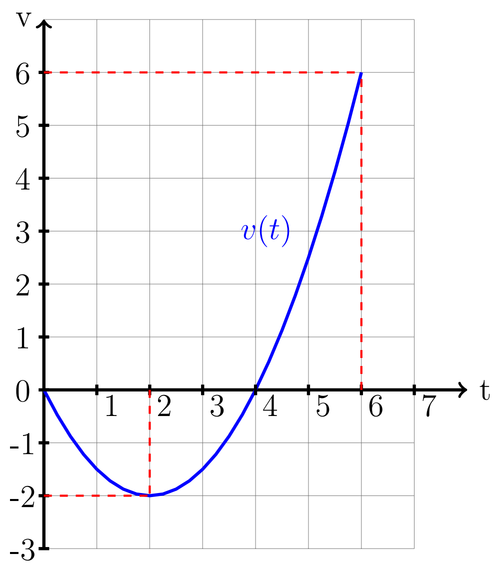
Q9[*Sec2.7, Rates of Change/Functions of motion*] A particle moves according to the law of motion $s(t) = t^3 - 5t^2 + 6t$, where t is measured in seconds and s in feet

(a)[1.4, Average velocity] Find the average velocity over the interval $[0, 2]$.

(b)[Velocity and position] Find the velocity $v(t)$ at time t.

(c)[Acceleration and velocity] What is the acceleration $a(6)$ after 6 seconds?

Q10[*Sec2.7, Graph of the velocity*] The accompanying figure shows the velocity $v(t)$ of a particle moving on a horizontal coordinate line, for t in the closed interval $[0, 6]$.



(a) When does the particle move forward?

(b) When does the particle slow down?

(c) When is the particle's acceleration positive?

(d) When does the particle move at its greatest speed in $[0, 6]$?

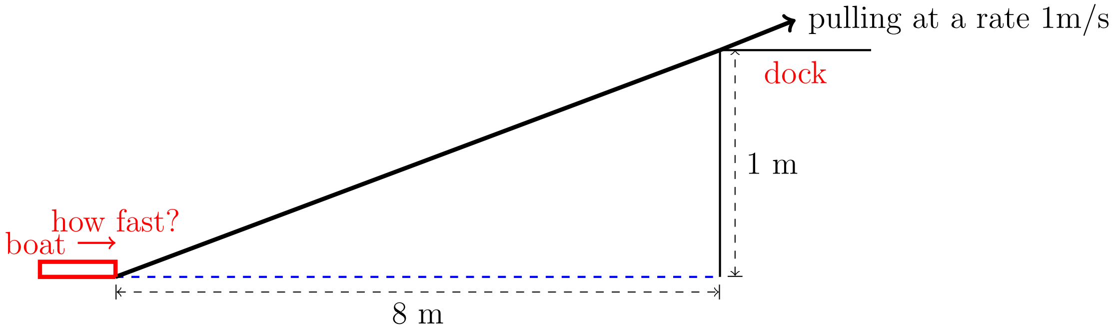
Q11[*Sec2.6, Implicit differentiation*] Consider the curve $y^2 + 2xy + x^3 = x$

(a) Find $\frac{dy}{dx}$ in terms of x, y.

(b) Find $\frac{dy}{dx}$ at $x = 1$ and find the slope of the tangent line of the curve at the point $(1, -2)$.

(c) Find the equation of the tangent line of the curve at the point $(1, -2)$.

Q12, *Sec2.8, Related Rates* A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8 m from the dock?



Algebraic

- $a^2 - b^2 = (a-b)(a+b)$
- $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Geometric

- Area of Circle: πr^2
- Circumference of Circle: $2\pi r$
- Circle with center (h,k) and radius r:
$$(x-h)^2 + (y-k)^2 = r^2$$
- Distance from (x_1, y_1) to (x_2, y_2):
$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$
- Area of Triangle: $\frac{1}{2}bh$
- $\sin\theta = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos\theta = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan\theta = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$
- If $\triangle ABC$ is similar to $\triangle DEF$ then
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
- Volume of Sphere: $\frac{4}{3}\pi r^3$
- Surface Area of Sphere: $4\pi r^2$
- Volume of Cylinder/Prism: (height)(area of base)
- Volume of Cone/Pyramid: $\frac{1}{3}$(height)(area of base)

Theorems

- (IVT) If f is continuous on $[a,b]$, $f(a) \neq f(b)$, and N is between $f(a)$ and $f(b)$ then there exists $c \in (a,b)$ that satisfies $f(c) = N$.

Limits

- $\lim_{x\to a} f(x)$ exists if and only if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$
- $\lim_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$
- $\lim_{\theta\to 0} \dfrac{1-\cos\theta}{\theta} = 0$

Derivatives

- $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$
- $(\cot x)' = -\csc^2 x$
- $(\csc x)' = -\csc x \cdot \cot x$

Trigonometric

- $\sin^2\theta + \cos^2\theta = 1$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
  $= 1 - 2\sin^2\theta$
  $= 2\cos^2\theta - 1$
- Table of Trig Values

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(x)$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos(x)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan(x)$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	DNE