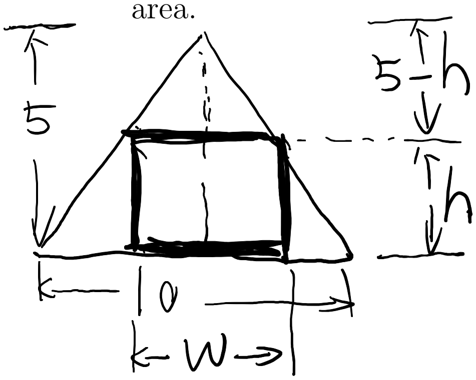
Name: ____________________

Complete the problems for 20 points. The scores will be recorded as Quiz9 and Quiz10 (10 pts each).

1. (6 points) Give an isosceles triangle with base 10cm and height 5cm. A rectangle is inscribed with its base on the base of the isosceles triangle and its upper corners on the two legs (the two two equal sides). What are the dimensions of such a rectangle with the greatest possible area? Find the greatest possible area.



$A = h \cdot w$

Similar triangles: $\frac{5-h}{5} = \frac{w}{10}$

$\Rightarrow w = \frac{5-h}{5} \times 10 = (5-h)\times 2$
$= 10 - 2h$.

$A = h \cdot (10-2h) = 10h - 2h^2$, h in $[0, 5]$

$A' = (10h - 2h^2)' = 10 - 2\cdot 2h = 10 - 4h = 0$. $h = \frac{10}{4} = \frac{5}{2}$

Largest area $A = w \cdot h = 5 \times \frac{5}{2} = \frac{25}{2}$. $w = 10 - 2\times\frac{5}{2} = 5$

2. (2 points) Find the most general anti-derivative of

$$f(x) = 4\cos x + 8 - \frac{\tan x \sec x}{3}.$$

Anti-D: $\boxed{4 \cdot \sin x + 8x - \frac{1}{3} \cdot \sec x + C.}$

3. (2 points) Evaluate

$$\int \frac{x^2}{4} - \sqrt{x} - \sin x \, dx.$$

$= \int \frac{1}{4} x^2 - x^{\frac{1}{2}} - \sin x \, dx.$

$= \boxed{\frac{1}{4} \cdot \frac{1}{2+1} x^{2+1} - \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} - (-\cos x) + C}$

4. (3 points) A car is moving along x-axis with position $x(t)$, velocity $v(t)$ at time t. Find $x(3)$ if $v(t) = -2t + 1$ ft/s^2 and $x(0) = 5$ ft.

position is the antiD of velocity.

$v(t) = -2t + 1$

$\Rightarrow x(t) = -2 \times \frac{1}{2} t^2 + t + C$

$= -t^2 + t + C.$

$x(0) = 0 + C = 5.$

$x(t) = -t^2 + t + 5.$

$\Rightarrow x(3) = -9 + 3 + 5 = -1$ ft.

5. (4 points) Estimate the area under the graph of $f(x) = 28 + 12x - x^2$ from $x = -2$ to $x = 14$. Find the estimate of the area using 4 rectangles of equal width and left endpoints. (Do not need to simplify. Just list the sum.)

$\Delta x = \frac{14-(-2)}{4} = \frac{16}{4} = 4.$

x: -2 2 6 10 14.

$f(-2) = 28 + 12\cdot(-2) - (-2)^2 = 0$

$f(2) = 28 + 12\cdot 2 - 2^2 = 48$

$f(6) = 28 + 12\cdot 6 - 6^2 = 64$

$f(10) = 28 + 12\cdot 10 - 10^2 = 48$

f(x). 0 48 64 48

Left-Sum $= \boxed{4\cdot(0+48+64+48)}$

6. (3 points) The limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \left(3 + \frac{i}{n} - 4\left(\frac{i}{n}\right)^2\right) = \sum_{i=1}^{n} \frac{1}{n}\, 2\left(3 + \frac{i}{n} - 4\left(\frac{i}{n}\right)^2\right).$$

is the limit of a Riemann sum for a certain definite integral $\int_a^b f(x)\,dx$. Find the expression of the integral. (Find a, b and $f(x)$. DO NOT NEED to EVALUATE.)

$\int_0^1 2(3 + x - 4x^2)\,dx.$

0 1

$\Delta x = \frac{1}{n}.$

a b.

$x_i = \frac{i}{n}.$