Name: ____

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish the test for 10 points.

•(MVT) If f is continuous on $[a,b]$ and differentiable on (a,b) then there exists $c \in (a,b)$ that satisfies $f'(c) = \frac{f(b)-f(a)}{b-a}$

1. (4 points) If the Mean Value Theorem is applied to the function $f(x) = x^2 + x$ on the interval $[0,3]$, what value of c satisfies the conclusion of the theorem in this case? (Find the value of c and show your work.)

$f'(x) = 2x+1 \qquad f(3) = 3^2+3 = 9+3 = 12, \quad f(0) = 0^2+0 = 0$

$$2c+1 = \frac{f(3)-f(0)}{3-0} = \frac{12-0}{3} = 4$$

$$2c+1 = 4 \Rightarrow 2c = 3$$

$$\Rightarrow \boxed{c = \frac{3}{2}}$$

2. Suppose $f(x) = x^3 - 3x^2$.

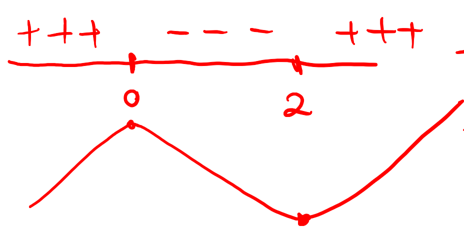
(a) (2 points) Compute $f'(x)$ and find all x such that $f'(x) = 0$.

$$f'(x) = 3x^2 - 6x = 3x(x-2) = 0$$

$$\Rightarrow \boxed{x=0} \text{ and } \boxed{x=2}$$

(b) (2 points) Find the intervals where $f(x)$ is increasing and decreasing.

$f' = 3x(x-2)$



$f'>0$ increasing $(-\infty, 0) \cup (2, +\infty)$

$f'<0$ decreasing $(0, 2)$

(c) (2 points) Find all values of x where $f(x)$ attains its local maximum or minimum.

local max at $x=0$

local min at $x=2$

(★ Finish the problems on the front page first. No more than 10 points may be earned on the quiz. The extra problem is of average (or above) actual exam difficulty level. It is recommended to do it now or later to check whether you handle the materials well enough for the exam.)

[2 extra points] Identify the intervals over which $f(x) = x^3 - 3x^2$ is concave up and down, and find all values of x where $f(x)$ has an inflection point.

$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6 = 6(x-1) = 0$

$x = 1$

f'' $--$ $++$

1

$f'' > 0$ concave up $(1, \infty)$

$f'' < 0$ concave down $(-\infty, 1)$