Name: ____

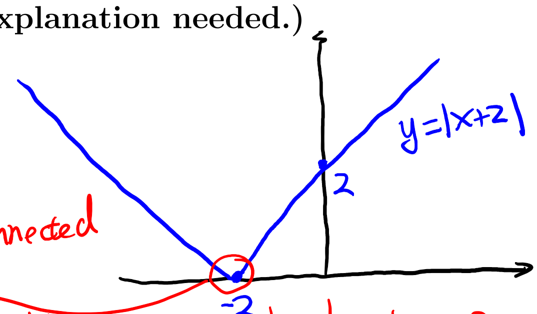
Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish the test for 10 points.

The one on the back worth two extra points. Maximum of 10 points will be recorded for each quiz.

Formulas:

- $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$
- $a^2 - b^2 = (a+b)(a-b)$

1. Let $f(x) = |x+2|$. Answer the following questions. (No explanation needed.)



(a) (1 point) Is $f(x)$ continuous at $x = 0$?

Yes

(b) (1 point) Is $f(x)$ continuous at $x = -2$?

Yes. The graph is connected at x=-2

There is no jump or break at x=-2.

(c) (1 point) Is $f(x)$ differentiable at $x = -2$?

No. The graph has a sharp turn at x=-2.

2. (4 points) Use THE LIMIT DEFINITION OF DERIVATIVE to find the derivative function $f'(x)$ of $f(x) = \sqrt{x-1}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h-1} - \sqrt{x-1}}{h}$$

$$= \lim_{h\to 0} \frac{(\sqrt{x+h-1} - \sqrt{x-1})(\sqrt{x+h-1} + \sqrt{x-1})}{h \cdot (\sqrt{x+h-1} + \sqrt{x-1})}$$

$$= \lim_{h\to 0} \frac{x+h-1-(x-1)}{h\cdot(\sqrt{x+h-1} + \sqrt{x-1})}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-1} + \sqrt{x-1})}$$

Plug in h=0

$$\frac{1}{\sqrt{x+0-1} + \sqrt{x-1}} = \boxed{\frac{1}{2\sqrt{x-1}}}$$

Remark:

$$(\sqrt{x+h-1} - \sqrt{x-1})(\sqrt{x+h-1} + \sqrt{x-1})$$

$$= (\sqrt{x+h-1})^2 - (\sqrt{x-1})^2$$

$$= x+h-1-(x-1)$$

$$= \cancel{x} + h \cancel{-1} \cancel{-x} \cancel{+1}$$

$$= h.$$

3. (3 points) Find the equation of the tangent line of $f(x) = \sqrt{x-1}$ at the point $(2, 1)$. (Use the expression of the derivative you find in Problem 2.)

By 2. $f'(x) = (\sqrt{x-1})' = \dfrac{1}{2\sqrt{x-1}}$

Slope of the tangent line $= f'(2) = \dfrac{1}{2\sqrt{2-1}} = \dfrac{1}{2}$.

Point : $(2,1)$

Point-Slope formula: $\boxed{y = \frac{1}{2}(x-2)+1}$

$\Leftrightarrow y = \frac{1}{2}x$

- **(IVT) If f is continuous on $[a, b]$, $f(a) \neq f(b)$, and N is between $f(a)$ and $f(a)$ then there exists $c \in (a, b)$ that satisfies $f(c) = N$.**

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x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin x$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos x$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan x$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	**DNE**

(★ Finish the previous three problems first. No more than 10 points may be earned on the quiz. The extra problem is of average (or above) actual exam difficulty level. It is recommended to do it now or later to check whether you handle the materials well enough for the exam.)
[2 extra points] **Use IVT to prove that there is a solution to the equation**

$$3\cos x = 1.$$

Let $f(x) = 3\cos x$, which is continuous on $(-\infty, \infty)$.

$f(0) = 3\cdot\cos 0 = 3$, $f(\frac{\pi}{2}) = 3\cdot\cos\frac{\pi}{2} = 0$.

$N=1$ is between $f(0)=3$ and $f(\frac{\pi}{2})=0$. By IVT, there is a $c \in (0, \frac{\pi}{2})$ such that

$f(c) = 1$ i.e. $\boxed{3\cos c = 1}$

The answer is not unique. e.g. We can also consider the interval $[\frac{\pi}{3}, \frac{\pi}{2}]$.

$f(\frac{\pi}{3}) = 3\cdot\cos\frac{\pi}{3} = \frac{3}{2}$ and $f(\frac{\pi}{2}) = 0$. Then $c \in (\frac{\pi}{3}, \frac{\pi}{2})$. Caution $[0, \frac{\pi}{3}]$ does not work.

We can also consider $\cos x = \frac{1}{3}$ and the function $f(x) = \cos x$ instead.