1[*Sec2.9, Linear Approximation*]

- Linearization of f at a: $L(x) = f(a) + f'(a)(x - a)$

Q1.1. Use linearization to find a good approximation of $\sqrt{9.02}$.
Hint: consider the linearization formula for $f(x) = \sqrt{x}$ at $a = 9$.

Q1.2. The radius of a sphere was measured to be 10 cm with a possible error of $\frac{1}{2}$ cm. Use a differential to estimate the maximum error in the calculated (a) surface area; (b) volume.

2[*Sec3.1, Extreme Values*]

- ***Extremal Value Theorem***: If $f(x)$ is continuous on the closed, finite interval $x \in [a, b]$, then $f(x)$ possesses at least one maximum point and one minimum point.

- ***Critical points***: For a function $f(x)$, a critical point (or critical number) is a point $x = c$ where the derivative is either zero or the function is not differentiable: $f'(c) = 0$ or f' undefined

Q2.1 Find the absolute maximum value of $f(x) = 6\pi x - 3x^2$ on the interval $[0, 2\pi]$ and where the maximum is obtained.

Q2.2 Find all the extrema of $f(x) = \sin x + \cos x$ on the interval $[0, \pi]$ and where the extrema are obtained.

Q2.3 Find the critical numbers (i.e., critical points) of the following functions

$$f(x) = x^{7/4} + \frac{9}{x^{1/4}}$$

3[*Sec3.2, Mean Value Theorem*]

- ***(MVT)*** If f is continuous on $[a, b]$ and differentiable on (a, b) then there exists $c \in (a, b)$ that satisfies $f'(c) = \frac{f(b)-f(a)}{b-a}$

Q3: If the Mean Value Theorem is applied to the function $f(x) = x^2 - 2x$ on the interval $[1, 4]$, what value of c satisfies the conclusion of the theorem in this case?

4.1[*Sec3.3, Derivatives and Graphs*]

- ***Increasing/Decreasing Theorem***: Let $f(x)$ be continuous on $[a, b]$.
  - If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on $[a, b]$.
  - If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on $[a, b]$.
- ***Concavity Theorem***: Let $f(x)$ be a function.
  - If $f''(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is concave up over (a, b).
  - If $f''(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is concave down over (a, b).
  - If $f''(x) = 0$ and $f''(x)$ changes its sign at $x = c$, then $f(x)$ has an inflection point at $x = c$.

4.2[*Sec3.4, Limits at Infinity*]

- ***Vertical asymptote***: $x = a$ is a V.A. of $f(x)$ if $f(x) \to \pm\infty$ as $x \to a$.
- ***Horzontal asymptote***: $y = L$ is a H.A. of $f(x)$ if $f(x) \to L$(finite) as $x \to \pm\infty$
- ***Limit at infinity***:
  - ***Limit for power functions of*** x:

$$p > 0, \quad \lim_{x\to\pm\infty} x^{\mathrm{p}} = \pm\infty \text{ (the sign depends on } p), \quad \lim_{x\to\pm\infty} x^{-\mathrm{p}} = \lim_{x\to\pm\infty} \frac{1}{x^{\mathrm{p}}} = 0$$

  - ***The highest term rule:*** Keep the highest term **<u>in each brackets</u>** in the numerator and denominator. Drop all the lower order terms.

4.3[*Sec3.5, Curve Sketching*]

- ***Slant asymptote:*** If a rational function $f(x) = mx + b + \frac{r(x)}{d(x)}$ via polynomial long(short) division and
$$\lim_{x\to\pm\infty} f(x) - (mx + b) = \lim_{x\to\pm\infty} \frac{r(x)}{d(x)} = 0,$$
then $y = mx + b$ is a S.A. of $f(x)$
- ***Method for Graphing:***
  1. Determine the domain of $f(x)$. Find the x-intercepts (solve for $f(x) = 0$); and compute the y-intercept $f(0)$ if there are any(may be none).
  2. Determine the derivatives $f'(x)$, $f''(x)$ with Derivative Rules. Find all the increasing/decreasing and concave up/down intervals. Find all local max/min and inflection points if there are any.
  3. Find all vertical/horizontal/slant asymptotes.
  4. Draw all the above features on the graph.

Q4 : Find all vertical and horizontal asymptotes of
$$f(x) = \frac{3x^2 - 3}{x^2 + x - 6}$$

Q5: How many vertical and slant asymptote(s) does $y = f(x)$ have?
$$f(x) = \frac{x^2 - 8x + 9}{2x + 1}$$

Q6: Suppose

$$f(x) = \frac{3x^2}{(x+2)^2}, \quad f'(x) = \frac{12x}{(x+2)^3}, \quad f''(x) = -\frac{24(x-1)}{(x+2)^4}$$

Answer the following questions or enter none in the case of no answer.

(a) Find the x and y intercepts of $y = f(x)$.

(b) Find all the asymptotes of $y = f(x)$.

(c) Find all the critical points of $y = f(x)$.

(d) Find all the interval(s) where f is increasing and where f is decreasing.

(e) Find all the interval(s) where f is concave up and where f is concave down.

(d) Find the inflection point(s) of f.

(f) Sketch the graph of $y = f(x)$.

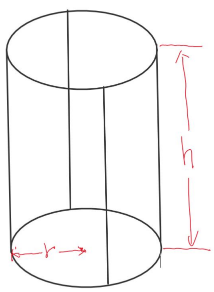
7[*Sec3.7, Optimization*]

1. Draw a picture labeled with all varying quantities. Find the target function which is to be maximized or minimized. Express the target function by other quantities.

2. Write equations relating variables. Choose one as the controlling variable, and solve for all other variables in terms of it. Plug into the target function and rewrite it using only one variable. ***Determine the domain.***

3. Find the absolute maximum/minimum of the target function.

Q7 Suppose we have 16 ft of steel wire to make a skeleton of a cylinder, with two circles (radius r) and 4 sides (height h). What is the largest possible surface area of the cylinder?



Q8[Newton's Method] $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ is used to approximate the root of the equation $f(x) = 0$. Newton's method can be used to approximate $\sqrt[4]{4}$ by finding the root of which of the following functions?

A. $f(x) = x - 4$. **B.** $f(x) = x^2 - 4$. **C.** $f(x) = x^4 - 4$. **D.** $f(x) = \sqrt[4]{x} - 4$.

Q9[*Sec3.9, Antiderivatives*]

- ***Antiderivative***. $F(x)$ is an antiderivative of $f(x)$ if $F'(x) = f(x)$. $F(x) + C$ for any constant C is called the most general antiderivative of $f(x)$
- $x^n = nx^{n-1}$, $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(\tan x)' = \sec^2 x$, $(\sec x)' = \sec x \cdot \tan x$
- ***Antiderivative Table:***

$f(x)$	$x^n, n \neq -1$	$\cos x$	$\sin x$	$\sec^2 x$	$\sec x \cdot \tan x$
Anti-D F(x)	$\frac{1}{n+1}x^{n+1}$	$\sin x$	$-\cos x$	$\tan x$	$\sec x$

- $f(x)$ is the (most general) anti-D of $f'(x)$. $f(a) = b$ can be used to determine the constant C.
- Position $s(t)$ is the anti-D of velocity $v(t)$. $v(t)$ is the anti-D of acceleration $a(t)$.

Q9.1: Evaluate

$$\int 2\sec^2(x) - \frac{\cos x}{5} + 8x \, \mathrm{d}x$$

Q9.2 : Solve the following initial value problem: Suppose $f'(x) = \sqrt{x}$ and $f(0) = 1$. Find $f(x)$.

Q9.3 A car traveling at 20ft/s decelerates at 4ft/s^2. Find the velocity function $v(t)$ at time t. Assume that initial position is $s(0) = 3$ ft, find the postion after 3 s.

10[*Sec4.1, Area and Distance*]

- Approximating the area under the curve by finite rectangles; Four types of sum: Left, Right, Upper(Overstimate) and Lower(Underestimate) sums.
- Area/Integral under $y = f(x)$ on $[a, b]$ as the limit of a Riemann sum.

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x,\ \ \Delta x = \frac{b-a}{n}, x_i = a + i\Delta x, i = 1, 2, \cdots, n$$

Q10.1: Find (a) the Left-endpoints sum and (b) the Right-endpoints sum, when we estimate the area under the graph of $f(x) = x^2 - 2x - 3$ from $x = 0$ to $x = 4$ using four rectangles of equal width.

11[*Sec4.2, The Definite Integral*]

- ***(Definite) Integral*** as ***Area under the curve*** and as ***the limit of a Riemann sum***

$$\int_a^b f(x)dx = \text{Area under } f(x)(\text{up to sign}) = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(a + i\frac{b-a}{n}\right) \cdot \frac{b-a}{n}$$

- Integral Rules.

Sum/Diff/Const.Multi.: $\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$; $\int_a^b C \cdot f(x)dx = C \cdot \int_a^b f(x)dx$

Splitting/Fliping:

$$\int_a^c f(x)dx = \int_a^{\boxed{b}} f(x)dx + \int_{\boxed{b}}^c f(x)dx;\quad \int_a^a f(x)dx = 0;\quad \int_a^b f(x)dx = -\int_b^a f(x)dx$$

- Basic integrals from the graph:

Rectangle: $\int_a^b 1dx = b - a$; $\int_a^b Cdx = C(b - a)$

Half/Quater disk: $\int_{-1}^1 \sqrt{1-x^2}dx = \frac{1}{2}\pi$; $\int_{-r}^r \sqrt{r^2-x^2}dx = \frac{1}{2}\pi r^2$; $\int_0^r \sqrt{r^2-x^2}dx = \frac{1}{4}\pi r^2$

Triangle/Trapezoid: $\int_0^b xdx = \frac{1}{2}b^2$; $\int_a^b xdx = \frac{1}{2}b^2 - \frac{1}{2}a^2$

Q11.1: Evaluate the limit of following Riemann sum

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \left(4 \cdot \frac{i}{n} - 3 \right)$$

Q11.2: Suppose $\int_2^5 f(x)\ dx = 3$ and $\int_2^3 f(x)\ dx = -4$. Find $\int_3^5 2f(x)\ dx$.

Q11.3: Evaluate (Hint: a definite integral represents an area.)

$$\int_0^3 \sqrt{9-x^2}dx, \quad \text{and} \quad \int_0^4 |3-x|dx$$

12[*Sec4.3, Fundamental Theorem of Calculus*]

- ***FToC P1:*** If $F(x) = \int_a^x f(t)\,dt$, then $F'(x) = \left(\int_a^x f(t)\,dt\right)' = f(x)$.

- ***FToC P1 Chain rule form:*** $\left(\int_{v(x)}^{u(x)} f(t)\,dt\right)' = f\big(u(x)\big)\cdot u'(x) - f\big(v(x)\big)\cdot v'(x)$

$$\left(\int_a^{u(x)} f(t)\,dt\right)' = f\big(u(x)\big)\cdot u'(x), \quad \left(\int_{v(x)}^{b} f(t)\,dt\right)' = -f\big(v(x)\big)\cdot v'(x)$$

- ***FToC P2:*** If $F(x)$ is an anti-D of $f(x)$, i.e., $F'(x) = f(x)$, then $\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a)$

- ***Antiderivative Table:***

$f(x)$	$x^n, n \neq -1$	$\cos x$	$\sin x$	$\sec^2 x$	$\sec x \cdot \tan x$
Anti-D F(x)	$\frac{1}{n+1}x^{n+1}$	$\sin x$	$-\cos x$	$\tan x$	$\sec x$

Q12.1: Let

$$F(x) = \int_2^{\cos x} \sqrt{5 - t^2}\,dt,$$

find $F'(x)$.

Q12.2: Evaluate

$$\int_1^2 \frac{5 - 7t^6}{t^4}\,dt$$

Algebraic

- $a^2 - b^2 = (a-b)(a+b)$
- $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Geometric

- Area of Circle: πr^2
- Circumference of Circle: $2\pi r$
- Circle with center (h,k) and radius r:
$$(x-h)^2 + (y-k)^2 = r^2$$
- Distance from (x_1, y_1) to (x_2, y_2):
$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$
- Area of Triangle: $\frac{1}{2}bh$
- $\sin\theta = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos\theta = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan\theta = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$
- If $\triangle ABC$ is similar to $\triangle DEF$ then
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
- Volume of Sphere: $\frac{4}{3}\pi r^3$
- Surface Area of Sphere: $4\pi r^2$
- Volume of Cylinder/Prism: (height)(area of base)
- Volume of Cone/Pyramid: $\frac{1}{3}$(height)(area of base)

Trigonometric

- $\sin^2\theta + \cos^2\theta = 1$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 1 - 2\sin^2\theta \\ &= 2\cos^2\theta - 1\end{aligned}$
- Table of Trig Values

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(x)$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos(x)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan(x)$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	DNE

Limits

- $\lim_{x\to a} f(x)$ exists if and only if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$
- $\lim_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$
- $\lim_{\theta\to 0} \dfrac{1-\cos\theta}{\theta} = 0$

Derivatives

- $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$
- $(\cot x)' = -\csc^2 x$
- $(\csc x)' = -\csc x \cdot \cot x$

Theorems

- (IVT) If f is continuous on $[a,b]$, $f(a) \neq f(b)$, and N is between $f(a)$ and $f(b)$ then there exists $c \in (a,b)$ that satisfies $f(c) = N$.
- (MVT) If f is continuous on $[a,b]$ and differentiable on (a,b) then there exists $c \in (a,b)$ that satisfies $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.
- (FToC P1) If $F(x) = \displaystyle\int_a^x f(t)\,dt$ then $F'(x) = f(x)$.

Other Formulas

- Newton's Method: $\quad x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
- $\displaystyle\sum_{i=1}^{n} c = cn$
- $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$