MTH 132, Practice Final, Sec39, Fall 2018

Multiple Choice Problems.

1. Suppose $f(x)$ is a continuous function with values given by the table below.

x	-2	-1	0	1
f(x)	0	3	0	-3

Which of the following statements are correct?

I $f(x) = 2$ must have a root $c \in (-1, 0)$.

II $f(x) = 2$ must have a root $c \in (0, 1)$.

III $f'(x) = 3$ must have a root $c \in (-1, 0)$.

IV $f'(x) = 0$ must have a root $c \in (-2, 0)$.

A. Only II; **B.** Only IV; **C.** Only I and IV; **D.** Only II and III; **E.** None of the above.

2. Suppose you are estimating the root of $x^3 = 5x - 1$ using Newton's method. If you use $x_1 = 2$, find the exact value of x_2

A $x_2 = 2 - \frac{1}{7}$

B $x_2 = 2 + \frac{1}{7}$

C $x_2 = 8 - \frac{8}{9}$

D $x_2 = 8 + \frac{8}{9}$

E $x_2 = 5 + \frac{1}{7}$

3. Consider the function

$$f(x) = \frac{2 - x^2}{x(x-3)}$$

Which of the following statements are correct?

I $f(x)$ has vertical asymptotes $y = 3$ and $y = 0$.

II $f(x)$ has vertical asymptotes $x = 0$ and $x = 3$.

III $f(x)$ has a horizontal asymptote $y = -1$.

IV $f(x)$ has no horizontal asymptote.

V $f(x)$ has no slant asymptote.

A. Only II; **B.** Only II and IV; **C.** Only I,III and V; **D.** Only II,III and V; **E.** None of the above.

4. Consider the function:

$$f(x) = x^3 + 6x$$

Which of the following statements are correct?

I $f(x)$ is an odd function.

II $f(x)$ is increasing for $x > 0$ and is decreasing on for $x < 0$.

III $f(x)$ is increasing on for all x.

IV $f(x)$ is concave up on $(0, \infty)$ and concave down on $(-\infty, 0)$.

V $f(x)$ has no critical point and no inflection point.

A. Only I, III, V; **B.** Only I,II, IV; **C.** Only I,III, IV; **D.** Only III, IV; **E.** None of the above.

5. Compute the limit:

$$\lim_{x \to 5} \frac{\sqrt[3]{x} - \sqrt[3]{5}}{x - 5}$$

A $+\infty$

B 0

C $\frac{1}{3}\sqrt[3]{5}$

D $\frac{1}{3}5^{-\frac{2}{3}}$

E Does not exist

6. Find the limit:

$$\lim_{x \to 0} 3x \cdot \sin(\frac{1}{2x})$$

A $\frac{2}{3}$

B $\frac{3}{2}$

C 0

D ∞

E Does not exist.

7. For what value of k will $f(x)$ be continuous for all values of x?

$$f(x) = \begin{cases} \frac{x^2-3k}{x-3} & \text{if} \quad x \leq 2; \\ \\ 8x - k & \text{if} \quad x > 2; \end{cases}$$

A $k = 2$

B $k = 3$

C $k = 4$

D $k = 5$

E None of the above

8. Evaluate

$$\int_{-\pi}^{\pi} \sin x \cdot \sqrt{\cos x + 2} \; dx$$

A $\frac{4}{3}$

B 0

C $-\frac{4}{3}$

D $-\frac{2}{3}$

E 2

9. Estimate the area under the graph of $f(x) = -x^2 + 4x + 5$ from $x = 0$ to $x = 4$ using the areas of 4. rectangles of equal width.

A The upper sum (overestimate) is 34 and the lower sum (underestimate) is 30.

B The upper sum (overestimate) is 34 and the lower sum (underestimate) is 26.

C The upper sum (overestimate) is 30 and the lower sum (underestimate) is 26.

D The upper sum (overestimate) is 17 and the lower sum (underestimate) is 13.

E None of the above.

10. A car is moving according to $s(t) = -t^2 + 4t + 5$. Which of the following statements are correct?

I The velocity at $t = 2$ is 4.

II The velocity at $t = 2$ is 0.

III The average velocity from $t = 0$ to $t = 2$ is 4.5.

IV The average velocity from $t = 0$ to $t = 2$ is 5.

A. Only II; **B.** Only III; **C.** Only I and IV; **D.** Only II and III; **E.** None of the above.

11. A car is moving according to $s(t) = -t^2 + 4t + 5$. Which of the following statements are correct?

I The car is slowing down from $t = 0$ to $t = 2$.

II The car is speeding up from $t = 2$ to $t = 4$.

III The car is moving in positive direction from $t = 0$ to $t = 5$.

IV The car is moving in negative direction from $t = 2$ to $t = 5$.

A. Only I; **B.** Only III; **C.**Only I,II and IV; **D.** Only I and II; **E.** None of the above.

12. Find y if

$$y' = 2x\sin(x^2), \quad y(0) = 4$$

A $y = -2\cos(x) + 6$

B $y = -\cos(x^2) + 5$

C $y = \cos(x^2) + 3$

D $y = x^2\cos(\frac{x^3}{3}) + 4$

E $y = x^2\sin(\frac{x^3}{3}) + 4$

Standard Response Problems.

1. Find $\frac{dy}{dx}$ if

   (a) $y = x\tan(x^2)$

   (b) $x^2 = 3y + \cos(y)$

2. If the radius of a circular ink blot is growing at a rate of 3 cm/min. How fast (in cm^2/min) is the area of the blot growing when the radius is 10 cm?

3. Let

$$f(x) = \frac{1}{2x - x^2} \qquad \text{for} \quad x \in (0, 2).$$

Let $f(0) = f(2) = 3$.

(a) Is $f(x)$ continuous on $[0, 2]$?

(b) Find the critical point and the local minimum of $f(x)$ in $(0, 2)$.

(c) Does $f(x)$ have absolute maximum or minimum in $[0, 2]$.

4. Evaluate

$$\int \frac{x^2}{\sqrt{3 + x^3}}\, dx$$

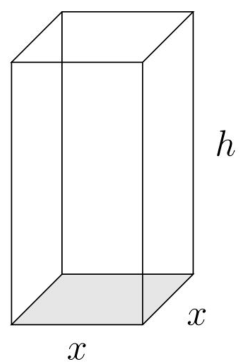
5. Suppose $f(x) = x^4 - 6x^2 - 3$.

(a) Identify the intervals over which $f(x)$ is increasing and decreasing, and all values of x where $f(x)$ attains its local maximum or minimum.

(b) Identify the intervals over which $f(x)$ is concave up and down, and all values of x where $f(x)$ has an inflection point.

(c) Sketch the graph of $y = f(x)$.

6. A box with square base and open top is to have a volume of 32 in^3. Find the dimensions of the box that minimizes the amount of material used.



7. Find the average of the function

$$f(x) = \frac{\cos x}{\sin^2 x}$$

over the interval $[\frac{\pi}{4}, \frac{\pi}{2}]$.

8. Find the area of the region enclosed by the graphs of the equations $y = -x - 4$ and $y = -x^2 + x + 4$.

Algebraic

- $a^2 - b^2 = (a-b)(a+b)$
- $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Geometric

- Area of Circle: πr^2
- Circumference of Circle: $2\pi r$
- Circle with center (h,k) and radius r:
$$(x-h)^2 + (y-k)^2 = r^2$$
- Distance from (x_1, y_1) to (x_2, y_2):
$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$
- Area of Triangle: $\frac{1}{2}bh$
- $\sin\theta = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos\theta = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan\theta = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$
- If $\triangle ABC$ is similar to $\triangle DEF$ then
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
- Volume of Sphere: $\frac{4}{3}\pi r^3$
- Surface Area of Sphere: $4\pi r^2$
- Volume of Cylinder/Prism: (height)(area of base)
- Volume of Cone/Pyramid: $\frac{1}{3}$(height)(area of base)

Trigonometric

- $\sin^2\theta + \cos^2\theta = 1$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$
- Table of Trig Values

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(x)$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos(x)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan(x)$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	DNE

Limits

- $\lim\limits_{x\to a} f(x)$ exists if and only if $\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x)$
- $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$
- $\lim\limits_{\theta\to 0} \dfrac{1-\cos\theta}{\theta} = 0$

Derivatives

- $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$
- $(\cot x)' = -\csc^2 x$
- $(\csc x)' = -\csc x \cdot \cot x$

Theorems

- (IVT) If f is continuous on $[a,b]$, $f(a) \neq f(b)$, and N is between $f(a)$ and $f(b)$ then there exists $c \in (a,b)$ that satisfies $f(c) = N$.
- (MVT) If f is continuous on $[a,b]$ and differentiable on (a,b) then there exists $c \in (a,b)$ that satisfies $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.
- (FToC P1) If $F(x) = \displaystyle\int_a^x f(t)\ dt$ then $F'(x) = f(x)$.

Other Formulas

- Newton's Method: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
- $\displaystyle\sum_{i=1}^{n} c = cn$
- $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$