§4.4, 5.5 More integrals

① Indefinite Integral of f: $\int f(x)\,dx = F(x) + C$, where $F'(x) = f(x)$.

- We want a new notation for the most general anti-D of $f(x)$,

  $\int f(x)\,dx = F(x) + C$, where $F(x)$ is any anti-D of $f(x)$, i.e., $F'(x) = f(x)$.

- In particularly, $\int F'(x)\,dx = F(x) + C$, and $F(x) = \int F'(x)\,dx \ (+C)$

☆ • Most important integrals: $\int x^n\,dx = \frac{1}{n+1} x^{n+1} + C$, $n \neq -1$. $\boxed{\int 1\,dx = x + C}$

☆ $\int \sin x\,dx = -\cos x + C$, $\int \cos x\,dx = \sin x + C$, $\int \sec^2 x\,dx = \tan x + C$, $\int \tan x \sec x\,dx = \sec x + C$.

eg.1. Find the most general function F such that $F'(x) = \frac{4}{\sqrt[3]{x}} - 2\tan x \sec x$.

s/n: $F(x) = \int \frac{4}{\sqrt[3]{x}} - 2\tan x \sec x\,dx$

$= \int 4 \cdot x^{-\frac{1}{3}} - 2\tan x \cdot \sec x\,dx$ Apply the formula of $\int x^n\,dx$ with $n = -\frac{1}{3}$

$= 4 \cdot \int x^{-\frac{1}{3}}\,dx - 2 \cdot \int \tan x \sec x\,dx$

$= \boxed{4 \cdot \frac{1}{-\frac{1}{3}+1} \cdot x^{-\frac{1}{3}+1} - 2 \cdot \sec x + C} = \frac{2}{3} x^{\frac{2}{3}} - 2 \cdot \sec x + C$

eg2. Evaluate the indefinite integral

$\int u \cdot (\sqrt{u} + 2)\,du = \int u \cdot u^{\frac{1}{2}} + 2u\,du$

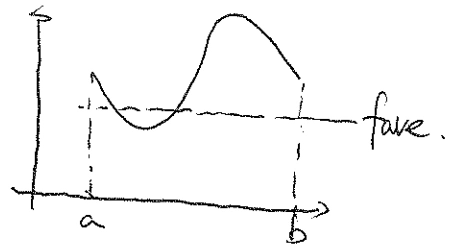
$= \int u^{\frac{3}{2}} + 2 \cdot u\,du$ Apply the formula with $n = \frac{3}{2}$ and $n = 1$

$= \frac{1}{\frac{3}{2}+1} u^{\frac{3}{2}+1} + 2 \cdot \frac{1}{1+1} u^{1+1} + C$

$= \boxed{\frac{2}{5} u^{\frac{5}{2}} + u^2 + C}$

- Average of $f(x)$ over $[a,b]$

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$



eg.4 Find the average of $f(x) = \frac{1}{x^2}$ on the interval $[1,4]$

sln: $f_{ave} = \frac{1}{4-1}\int_1^4 \frac{1}{x^2}dx = \frac{1}{3}\int_1^4 \frac{1}{x^2}dx = \frac{1}{3}\cdot\frac{3}{4} = \boxed{\frac{1}{4}}$

$$\int_1^4 \frac{1}{x^2}dx = \int_1^4 x^{-2}dx \underset{n=-2}{=} \frac{1}{-2+1}\cdot x^{-2+1}\Big|_1^4 = \frac{1}{-1}\cdot x^{-1}\Big|_1^4$$

$$= -1\cdot\frac{1}{x}\Big|_1^4$$

$$= -\frac{1}{4} - \left(-\frac{1}{1}\right) = -\frac{1}{4}+1 = \boxed{\frac{3}{4}}$$

- Mean Value Theorem for Integral.

If $f(x)$ is continuous on $[a,b]$, then there is some $c\in(a,b)$ such that

$$f(c) = f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

Rmk: The above formula can be derived from MVT and FTC2.

eg5. For $f(x) = \frac{1}{x^2}$ on $[1,4]$ in eg.4, find the value of $c\in(1,4)$ such that $f(c) = f_{ave}$.

sln: From eg 4, we know that $f_{ave} = \frac{1}{4}$

Then set $f(c) = \frac{1}{c^2} = \frac{1}{4}$, solve for c.

$$\Rightarrow 4 = c^2 \Rightarrow c = \pm 2.$$

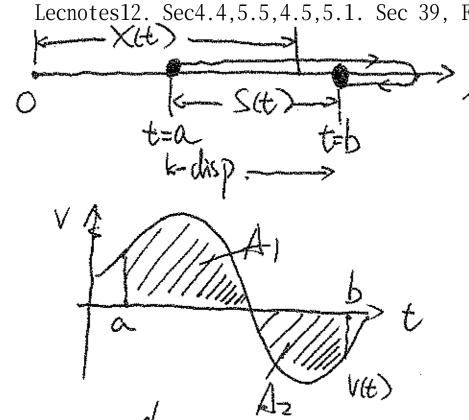
Only $\boxed{c=2}$ is in $(1,4)$.

② Moving particle with velocity $V(t)$.

Position function $x(t) = \int V(t)\,dt$

★ Displacement from $t=a$ to $t=b$ D.P. $= \int_a^b V(t)\,dt$

★ Distance from $t=a$ to $t=b$ D.T. $= \int_a^b |V(t)|\,dt$



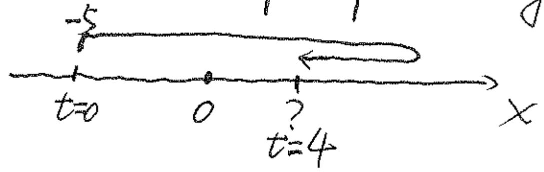
$\text{disp} = \int_a^b V\,dt = A_1 - A_2$

$\text{distance} = \int_a^b |V|\,dt = A_1 + A_2$.

eg3. An object moves along a coordinate (to the right) line with velocity $V(t) = t(2-t)$.

Its initial position is 5 units to the left of the origin.

(a) Find its position after 4 seconds.



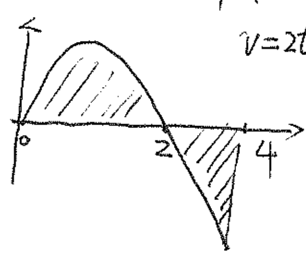
Displacement from $t=0$ to $t=4$.

$$\int_0^4 t(2-t)\,dt = \int_0^4 2t - t^2\,dt = t^2 - \frac{1}{3}t^3 \Big|_0^4 = (4^2 - \frac{1}{3}4^3) - 0 = 16 - \frac{64}{3}$$

Position = Initial position + Displacement $= -5 + 16 - \frac{64}{3} = \boxed{11 - \frac{64}{3}}$

★ (b). Find the distance traveled from $t=0$ to $t=4$.

$$\text{Distance} = \int_0^4 |t(2-t)|\,dt$$



$v = 2t - t^2 > 0 \quad 0 < t < 2$

$V < 0 \quad 2 < t < 4$.

$$= \int_0^2 |2t - t^2|\,dt + \int_2^4 |2t - t^2|\,dt$$

$$|V| = |2t - t^2| = \begin{cases} 2t - t^2 & 0 < t < 2 \\ t^2 - 2t & 2 < t < 4 \end{cases}$$

$$= \int_0^2 2t - t^2\,dt + \int_2^4 -(2t - t^2)\,dt$$

$$= t^2 - \frac{1}{3}t^3 \Big|_0^2 + (-t^2 + \frac{1}{3}t^3)\Big|_2^4$$

$$= 2^2 - \frac{1}{3}\cdot 2^3 - 0 + (-4^2 + \frac{1}{3}4^3) - (-2^2 + \frac{1}{3}\cdot 2^3)$$

$$= \frac{48}{3} - 8 = \boxed{8}$$

§ 4.5. Substitution Method

Key points: ① Differential Notation: If $u=g(x)$, then $du = g'(x)\cdot dx$.

② u-Sub: $\int f(\underbrace{g(x)}_{u})\cdot \underbrace{g'(x)\cdot dx}_{du} \xlongequal[du=g'(x)dx]{u=g(x)} \int f(u)\cdot du$

• Goal of u-Substitution Method: By changing the variable x into u, we convert the integral into one of those five basic integrals which we can deal with.

• Basic integrals: $\int x^n dx = \frac{1}{n+1}x^{n+1}+C$, $\int \sin x\, dx = -\cos x + C$, $\int \cos x\, dx = \sin x + C$

$\int \sec^2 x\, dx = \tan x + C$, $\int \sec x\cdot \tan x\, dx = \sec x + C$.

① Differential Notation and Composition of functions.

eg.1. $u = x^2+1$, $\frac{du}{dx} = (x^2+1)' = 2x \Rightarrow \boxed{du = 2x\cdot dx.}$

eg.2. If $f(x) = \sqrt{x}$, ~~then~~ $u = x^2+1$, then $f(u) = \sqrt{u} = \sqrt{x^2+1}$

② u-Sub method for indefinite integral

eg.3. Evaluate $\int \sqrt{u}\cdot du \xlongequal{n=\frac{1}{2}} \frac{1}{\frac{1}{2}+1}\cdot u^{\frac{1}{2}+1} + C = \boxed{\frac{2}{3}u^{\frac{3}{2}}+C}$

eg.4. $\int \sqrt{x^2+1}\cdot 2x\cdot dx$.

Set $u = x^2+1$, According to eg 1. $du = 2x\cdot dx$.

$= \int \sqrt{u}\cdot du$.

Plug in u and du. By substituting u and du the integral of x turns into an integral of u with simpler form, which we evaluate in eg 3.

$= \frac{2}{3}u^{\frac{3}{2}} + C$

$= \frac{2}{3}(x^2+1)^{\frac{3}{2}} + C$.

Replace u by x^2+1 in the last step.

• The key of u-sub is to find the right substitution u.

More examples:

e.g.5. Evaluate $\int x\cdot(2x^2+1)^3\cdot dx$. Hint: $u=2x^2+1$ is the ugly part.
(14 Final)

$u=2x^2+1$, $du=4x\cdot dx \Rightarrow \boxed{x\cdot dx=\frac{1}{4}\cdot du}$

$$=\int (2x^2+1)^3\cdot x\cdot dx$$
$$=\int u^3\cdot\frac{1}{4}\cdot du=\frac{1}{4}\cdot\frac{1}{3+1}\cdot u^{3+1}+C$$
$$=\frac{1}{16}\cdot u^4+C=\boxed{\frac{1}{16}(2x^2+1)^4+C}$$

e.g.6. Evaluate $\int\frac{\sec(\frac{x}{2})\cdot\tan(\frac{x}{2})}{\sqrt{\sec(\frac{x}{2})}}\cdot dx$. Hint: $u=\sec(\frac{x}{2})$ is the ugly part.
(14 Final)

$u=\sec(\frac{x}{2})$, $du=\sec(\frac{x}{2})\tan(\frac{x}{2})\cdot\frac{1}{2}\cdot dx$. chain rule for $(\sec\blacksquare)'=\sec\blacksquare\cdot\tan\blacksquare$

$$\Rightarrow 2du=\sec(\tfrac{x}{2})\cdot\tan(\tfrac{x}{2})\cdot dx$$

$$=\int\frac{2du}{\sqrt{u}}=\int 2\cdot u^{-\frac{1}{2}}\,du$$
$$=2\cdot\frac{1}{-\frac{1}{2}+1}\cdot u^{-\frac{1}{2}+1}+C$$
$$=2\cdot 2\cdot u^{\frac{1}{2}}+C \xrightarrow{\text{back to } x} \boxed{4\left(\sec(\tfrac{x}{2})\right)^{\frac{1}{2}}+C}$$

• Linear substitution and more general form of the five basic integrals.

☆ $\int(ax+b)^n=\frac{1}{n+1}\cdot x^{n+1}+C$, $\int\sin(ax+b)dx=\frac{-1}{a}\cos(ax+b)+C$, $\int\cos(ax+b)dx=\frac{1}{a}\sin(ax+b)+C$

$\int\tan(ax+b)\cdot\sec(ax+b)\cdot dx=\frac{1}{a}\sec(ax+b)+C$, $\int\sec^2(ax+b)dx=\frac{1}{a}\cdot\tan(ax+b)+C$.

e.g.7. $\int\sin(1-2x)\cdot dx \xrightarrow[du=-2dx]{u=1-2x} \int\sin u\cdot\frac{du}{-2}=\frac{1}{-2}(-\cos u)+C=\boxed{\frac{-1}{-2}\cos(1-2x)+C}$

$\frac{du}{-2}=dx$

③ u-sub for definite integral.

eg.8. Evaluate the definite integral $\int_0^2 \frac{1}{\sqrt{9-4x}}\,dx$.

Ugly part: $9-4x$. $u=9-4x$, $du=-4\cdot dx \Rightarrow dx=\frac{du}{-4}$

Caution: $\int_0^2$ 0, 2 are for x. They also change as we substitute $9-4x$ by u.

$$\int_{x=0}^{x=2} \xrightarrow{u=9-4x} \int_{u=9-4\cdot 0=9}^{u=9-4\cdot 2=1},\quad \int_{u=9}^{u=1}$$

$$\int_0^2 \frac{1}{\sqrt{9-4x}}\,dx \overset{u=9-4x}{=\!=} \int_9^1 \frac{1}{\sqrt{u}}\cdot\frac{du}{-4}$$ Use the flipping trick

$$= \int_1^9 \frac{1}{4}\cdot u^{-\frac{1}{2}}\,du = \frac{1}{4}\cdot\frac{1}{\frac{1}{2}}u^{\frac{1}{2}}\Big|_1^9$$

$$= \frac{1}{2}\cdot\sqrt{u}\Big|_1^9 = \boxed{\frac{1}{2}\cdot\sqrt{9}-\frac{1}{2}\cdot\sqrt{1}} = \boxed{1}$$

eg.9. Evaluate $\int_0^{\frac{\pi}{2}} 3\cdot\tan(\frac{x}{2})\cdot\sec^2(\frac{x}{2})\,dx$. Hint: $(\tan ■)' = \sec^2 ■$

$u=\tan(\frac{x}{2})$. $du=\frac{1}{2}\sec^2(\frac{x}{2})\,dx$.

$2du=\sec^2(\frac{x}{2})\,dx$.

$$\int_{x=0}^{x=\frac{\pi}{2}} \longrightarrow \int_{u=\tan 0=0}^{u=\tan(\frac{\pi}{4})=1}$$

$$= \int_0^1 3\cdot u\cdot 2du = 6\cdot\frac{1}{2}u^2\Big|_0^1 = \boxed{3\cdot 1^2-3\cdot 0^2=3}$$

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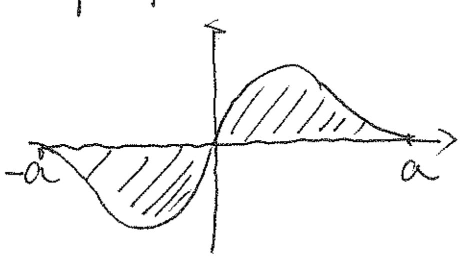
④ Symmetry integral.

If $f(x)$ is odd, i.e., $f(-x)=-f(x)$, then $\int_{-a}^{a} f(x)\,dx=0$



f odd means the graph of f is symmetric about the origin. Then the area above and below x-axis are the same, which will be cancelled out.

eg.10. $f(x)=\sin x\cdot(x^2+1)$. $\int_{-8}^{8}\sin x\cdot(x^2+1)\,dx=0$

since $f(-x)=\sin(-x)\cdot((-x)^2+1)=-\sin x\cdot(x^2+1)=-f(x)$ ($\sin x$ is odd)

More examples and hints for webwork:

eg 11. (ww8). (14 Final) $\int_8^{11} x\cdot\sqrt{x-7}\,dx$. Hint: $u=x-7$. $du=dx$.

$= \int x\cdot\sqrt{u}\,du$ — there is still one x left. keep substituting via the relation $u=x-7 \Leftrightarrow u+7=x$.

$= \int_1^4 (u+7)\cdot\sqrt{u}\cdot du$ — $x=11 \to u=x-7=4$, $x=8 \to u=x-7=1$

$= \int_1^4 u\cdot u^{\frac{1}{2}} + 7\cdot u^{\frac{1}{2}}\,du$

$= \int_1^4 u^{\frac{3}{2}} + 7\cdot u^{\frac{1}{2}}\,du = \frac{2}{5}u^{\frac{5}{2}} + 7\cdot\frac{2}{3}u^{\frac{3}{2}} \Big|_1^4$

$4^{\frac{5}{2}} = (\sqrt{4})^5 = 32$.

$4^{\frac{3}{2}} = (\sqrt{4})^3 = 8$

$= \frac{2}{5}\cdot 4^{\frac{5}{2}} + \frac{14}{3}\cdot 4^{\frac{3}{2}} - \left(\frac{2}{5}\cdot 1 + \frac{14}{3}\cdot 1\right)$

$= \frac{64}{5} + \frac{112}{3} - \frac{2}{5} - \frac{14}{3} = \boxed{\frac{62}{5} + \frac{98}{3}}$

eg 12. (ww6) $\int \frac{1}{x^2}\sin\left(\frac{3}{x}\right)\cdot\cos\left(\frac{3}{x}\right)dx$. Hint: $u=\cos\left(\frac{3}{x}\right)$.

$= \int \cos\left(\frac{3}{x}\right)\cdot\frac{1}{x^2}\sin\left(\frac{3}{x}\right)dx$.

$du = -\sin\left(\frac{3}{x}\right)\cdot\left(\frac{3}{x}\right)'\,dx$. chain rule. inner $\frac{3}{x}=3x^{-1}$

$= -\sin\left(\frac{3}{x}\right)\cdot\frac{-3}{x^2}\,dx$ — $(3\cdot x^{-1})' = 3\cdot\frac{-1}{x^2}$

$= \sin\left(\frac{3}{x}\right)\cdot\frac{3}{x^2}\,dx$.

$\Rightarrow \frac{1}{3}du = \sin\left(\frac{3}{x}\right)\cdot\frac{1}{x^2}\,dx$

$= \int u\cdot\frac{1}{3}du$

$= \frac{1}{3}\cdot\frac{1}{2}u^2 + C$

$= \boxed{\frac{1}{6}\cdot\left(\cos\left(\frac{3}{x}\right)\right)^2 + C}$

eg 13. (14 Final) $\int \frac{x^3}{\sqrt{1+2\cdot x^4}}\,dx$. Ugly part: $u=1+2x^4$, $du=8\cdot x^3\,dx$

$\Rightarrow \frac{1}{8}du = x^3\,dx$

$= \int \frac{1}{\sqrt{1+2x^4}}\cdot x^3\,dx$

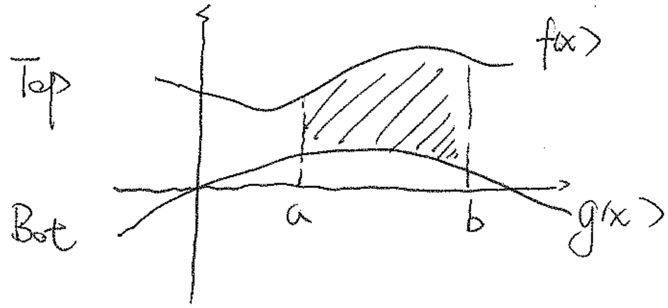
$= \int \frac{1}{\sqrt{u}}\cdot\frac{1}{8}du = \int u^{-\frac{1}{2}}\cdot\frac{1}{8}du = \frac{1}{-\frac{1}{2}+1}u^{-\frac{1}{2}+1}\cdot\frac{1}{8} + C$

$= 2(u)^{\frac{1}{2}}\cdot\frac{1}{8} + C$

$= \frac{1}{4}\cdot(1+2x^4)^{\frac{1}{2}} + C$.

§5.1 Area between curves

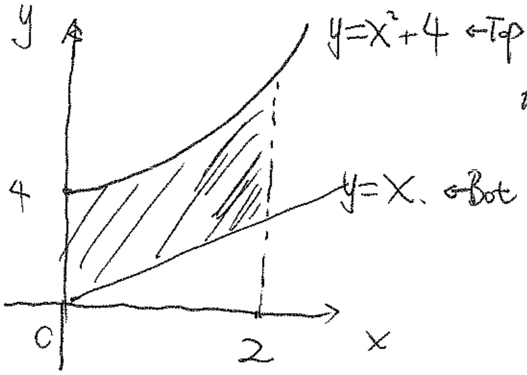
① Two curves : $y=f(x)$, $y=g(x)$ from $x=a$ to $x=b$



Area between the top curve and bot curve is given by

$$\int_a^b [f(x)-g(x)]\,dx.$$

e.g. 1. Sketch the region bounded by $y=x^2+4$, $y=x$, $x=0$, $x=2$ and find the area of the region.



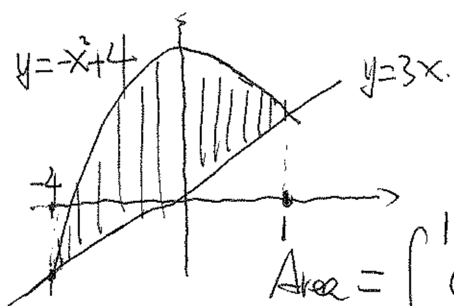
$$\text{Area} = \int_0^2 (x^2+4) - x\,dx.$$

$$= \tfrac{1}{3}x^3 + 4x - \tfrac{1}{2}x^2 \Big|_0^2$$

$$= \tfrac{1}{3}\cdot 2^3 + 4\cdot 2 - \tfrac{1}{2}2^2 - (0+0-0) = \boxed{\tfrac{8}{3}+6}$$

☆ eg. 2. sketch the region bounded by $y=-x^2+4$ and $y=3x$.

Find the intersections of the two curves and find the area of the region



Intersections: $-x^2+4=3x \Leftrightarrow x^2+3x-4=0$

$(x+4)(x-1)=0$

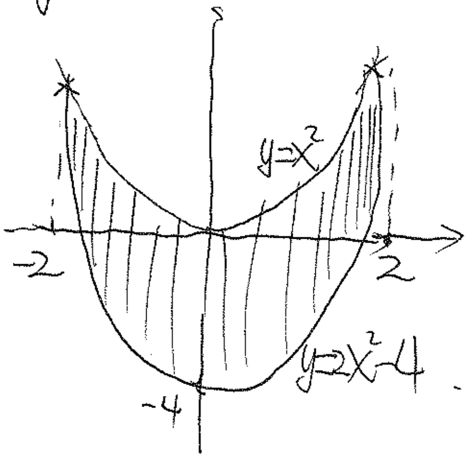
$\boxed{x=-4,\ x=1}$

$$\text{Area} = \int_{-4}^{1} (-x^2+4) - (3x)\,dx.$$

$$= -\tfrac{1}{3}x^3 + 4x - \tfrac{3}{2}x^2 \Big|_{-4}^{1} = -\tfrac{1}{3} + 4 - \tfrac{3}{2} - \left(-\tfrac{1}{3}(-4)^3 + 4(-4) - \tfrac{3}{2}\cdot(-4)^2\right)$$

$$= \boxed{-\tfrac{65}{3} + 44 - \tfrac{3}{2}}$$

eg3. Find the area of the region bounded by $y=x^2$, $y=2x^2-4$.



Intersections: $x^2 = 2x^2 - 4$

$\Leftrightarrow \quad x^2 = 4 \quad \Leftrightarrow \quad x = \pm 2$.

Top curve: $y = x^2$

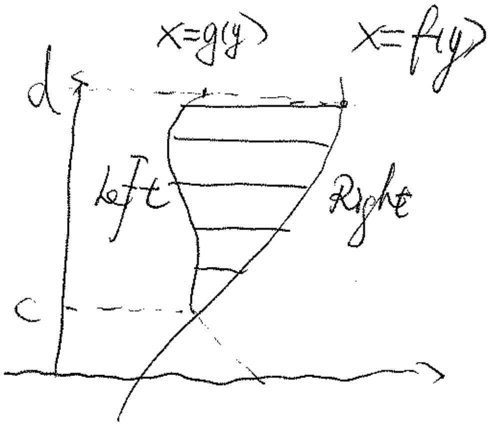
Bot curve: $y = 2x^2 - 4$

$$\text{Area} = \int_{-2}^{2} x^2 - (2x^2 - 4)\, dx.$$

$$= \int_{-2}^{2} -x^2 + 4\, dx = -\frac{1}{3}x^3 + 4x \Big|_{-2}^{2}$$

$$= -\frac{1}{3}2^3 + 4\cdot 2 - \left[-\frac{1}{3}(-2)^3 + 4(-2)\right]$$

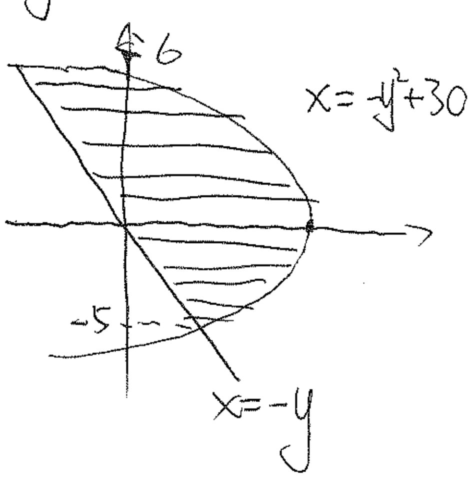
$$= -\frac{8}{3} + 8 - \frac{8}{3} + 8 = \boxed{16 - \frac{16}{3}}$$

② 

$$\text{Area} = \int_c^d \text{Right} - \text{Left}.\ dy$$

$$= \int_c^d f(y) - g(y)\, dy. \qquad (\text{ww}7\text{-}10).$$

eg4. (ww8): Find the area of the region bounded by $x = -y^2 + 30$, $x = -y$.



Intersections: $-y^2 + 30 = -y \Leftrightarrow y^2 - y - 30 = 0$

$y = 6,\ y = -5$.

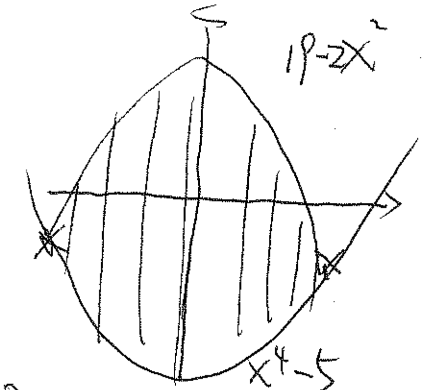
$$\text{Area} = \int_{-5}^{6} (-y^2 + 30) - (-y)\, dy$$

$$= -\frac{1}{3}y^3 + 30y + \frac{1}{2}y^2 \Big|_{-5}^{6} = \boxed{-\frac{341}{3} + 330 + \frac{11}{2}}$$

More hints on Webwork:

eg.5. (ww4).

sketch the region bounded by the curves $2x^2+y=19$ and $x^4-y=5$, then find the area of the region.



sln: $2x^2+y=19 \Rightarrow y=19-2x^2 \leftarrow$ Top

$x^4-y=5 \Rightarrow y=x^4-5 \leftarrow$ Bot.

Intersections: $19-2x^2=x^4-5$.

☆ $\Leftrightarrow x^4+2x^2-24=0 \Leftrightarrow (x^2+6)(x^2-4)=0$

$\Leftrightarrow x^2-4=0 \Leftrightarrow x=\pm 2$.

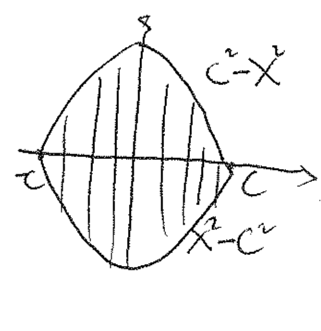
$$\text{Area}=\int_{-2}^{2} 19-2x^2-(x^4-5)\,dx=\int_{-2}^{2} 24-2x^2-x^4\,dx$$

$$=24x-\frac{2}{3}x^3-\frac{1}{5}x^5\Big|_{-2}^{2}=\boxed{\frac{1088}{15}}$$

eg.6 (ww6).

Find $c>0$ such that the area of the region bounded by $y=x^2-c^2$ and $y=c^2-x^2$ is 19

sln: Intersections: $x^2-c^2=c^2-x^2 \Rightarrow 2x^2=2c^2 \Rightarrow x^2=c^2 \Rightarrow x=\pm c$.



$$\text{Area}=\int_{-c}^{c} (c^2-x^2)-(x^2-c^2)\,dx$$

$$=\int_{-c}^{c} 2c^2-2x^2\,dx$$

$$=2c^2\cdot x-\frac{2}{3}\cdot x^3\Big|_{-c}^{c}$$

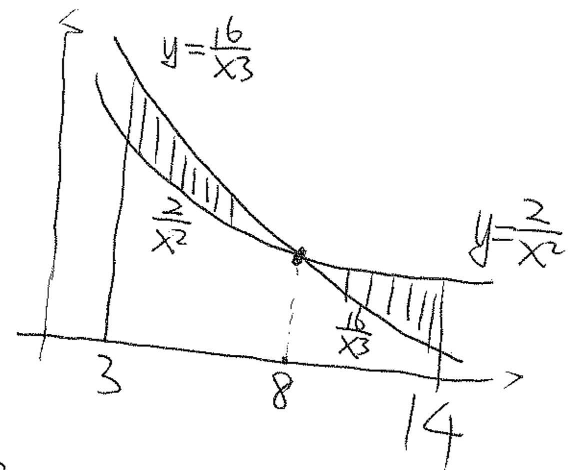
$$=2c^2\cdot c-\frac{2}{3}\cdot c^3-\left[2c^2\cdot(-c)-\frac{2}{3}\cdot(-c)^3\right]=\boxed{\frac{8}{3}c^3}$$

Set $\frac{8}{3}c^3=19 \Rightarrow c^3=\frac{3\cdot 19}{8} \Rightarrow \boxed{c=\left(\frac{3\cdot 19}{8}\right)^{\frac{1}{3}}}$

★★. eg 7 (ww 5)

Find the area of the region bounded by $y=\frac{16}{x^3}$, $y=\frac{2}{x^2}$, $x=3$, $x=14$

sln: Sketch the graph:



The two curves intersect at

$y=\frac{16}{x^3}=\frac{2}{x^2}$

$\Rightarrow 16x^2=2x^3$

$\Rightarrow \frac{16x^2}{2x^2}=\frac{2x^3}{2x^2}$

$\Rightarrow 8=x$

From $x=3$ to $x=8$, $y=\frac{16}{x^3}$ is the top curve
$y=\frac{2}{x^2}$ is the bot curve

From $x=8$ to $x=14$, $y=\frac{16}{x^3}$ is the bot curve
$y=\frac{2}{x^2}$ is the top curve

$$\text{Area}=\int_3^8 \frac{16}{x^3}-\frac{2}{x^2}\,dx+\int_8^{14}\frac{2}{x^2}-\frac{16}{x^3}\,dx.$$

$$=\int_3^8 16x^{-3}-2x^{-2}\,dx+\int_8^{14}2x^{-2}-16x^{-3}\,dx$$

$$=\left(16\cdot\frac{1}{-2}x^{-2}-2\cdot\frac{1}{-1}x^{-1}\right)\Big|_3^8+\left(2\cdot\frac{1}{-1}x^{-1}-16\cdot\frac{1}{-2}x^{-2}\right)\Big|_8^{14}$$

$$=\frac{16}{-2}\cdot 8^{-2}+2\cdot 8^{-1}-\left(\frac{16}{-2}\cdot 3^{-2}+2\cdot 3^{-1}\right)+\left(-2\cdot 14^{-1}+8\cdot 14^{-2}\right)-\left(-2\cdot 8^{-1}+8\cdot 8^{-2}\right)$$

$$=\boxed{-8\cdot\frac{1}{64}+2\cdot\frac{1}{8}+8\cdot\frac{1}{9}-2\cdot\frac{1}{3}+\frac{-2}{14}+\frac{8}{14^2}+2\cdot\frac{1}{8}-8\cdot\frac{1}{64}}$$