Name: _____ **ID:** _____

Clear your desk of everything except pens, pencils and erasers. Show all work clearly and in order. No notes, phones and calculators. You have 10 minutes to finish these THREE problems for 10 points.

Formula Sheet:

$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C, a \neq 1; \quad \frac{\mathrm{d}}{\mathrm{d}x}(\cosh x) = \sinh x; \quad \frac{\mathrm{d}}{\mathrm{d}x}(\sinh x) = \cosh x;$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \tan^{-1} x = \frac{1}{1+x^2}; \quad \sinh x = \frac{e^x - e^{-x}}{2}; \quad \cosh x = \frac{e^x + e^{-x}}{2}$$

1. (3 points) Find $y(x)$ if $y'(x) = \cosh x$ and $y(0) = 1$.

$y(x) = \int \cosh x \, dx = \sinh x + C$,

$y(0) = 1 \Rightarrow x = 0, y = 1 \Rightarrow 1 = \sinh 0 + C$, $\sinh 0 = \frac{e^0 - e^0}{2} = 0$

$1 = 0 + C \Rightarrow C = 1$.

$\boxed{y(x) = \sinh x + 1}$

2. (3 points) Find $f'(x)$ if $f(x) = 2^{\tan^{-1} x}$

Hint : $\int a^x dx = \frac{1}{\ln a} \cdot a^x + C \Rightarrow (a^x)' = \ln a \cdot a^x$

$f'(x) = \ln 2 \cdot 2^{\tan^{-1} x} \cdot (\tan^{-1} x)'$

$= \boxed{\ln 2 \cdot 2^{\tan^{-1} x} \cdot \frac{1}{1+x^2}}$

3. (4 points) Evaluate the following limit:

$$\lim_{x\to 0} \frac{x}{\tan(2x)}$$

$\lim_{x\to 0} \frac{x}{\tan(2x)}$ $\left(\frac{0}{\tan 0} = \frac{0}{0} \text{ case}\right)$

$\overset{\text{l'H}}{=} \lim_{x\to 0} \frac{(x)'}{(\tan(2x))'}$

$= \lim_{x\to 0} \frac{1}{\sec^2(2x)\cdot 2}$

$= \frac{1}{\sec^2 0 \cdot 2}$, $\sec 0 = \frac{1}{\cos 0} = 1$

$= \boxed{\frac{1}{2}}$