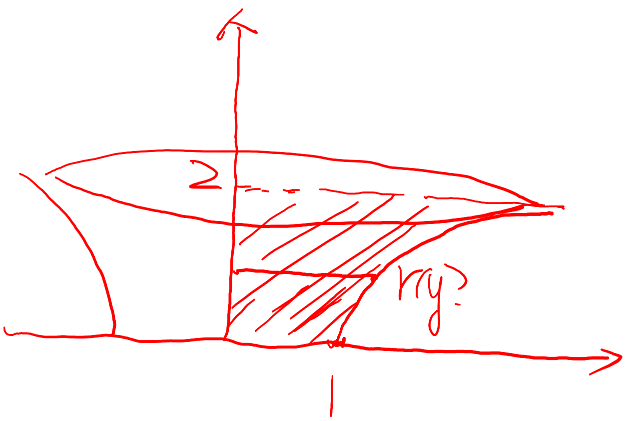
Hints and formulas

Practice Midterm1, Sec61, F17

Sec5.3. Volume. LecNote1.

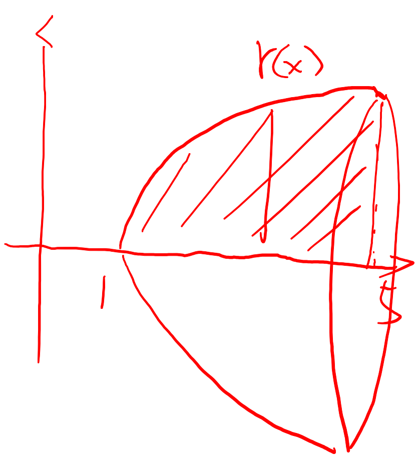
Q1 Find the volume of the following rotating solids.

(a)(Vertical Axe) The region R is bounded by $y = \sqrt{x-1}, y = 2, x = 0, y = 0$. The solid is generated by revolving the region R about the y axis. Sketch the region R and the rotating solid S. Find the volume of the rotating solid.



$$V = \int_a^2 \pi \cdot [r(y)]^2 \cdot dy$$

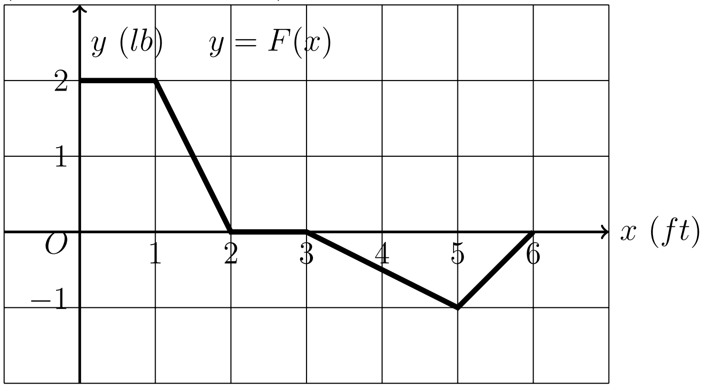
(b)(Horizontal Axe) The region R is bounded by $y = \sqrt{x-1}, y = 0, x = 5$. The solid is generated by revolving the region R about the axis $y = -1$. Sketch the region R and the rotating solid S. Find the volume of the rotating solid.



$$V = \int_1^5 \pi \cdot [r(x)]^2 dx$$

Sec5.4. Work. LecNote2.

Q2(Definition of Work.) Below is the graph of a force function $F(x)$ (in lbs).



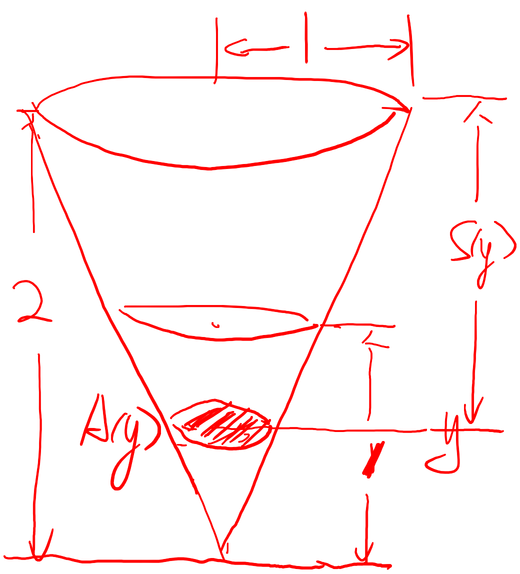
$$W = \int_a^b F(x)\,dx$$

Area "under" $F(x)$

(a) How much work is done by the force in moving an object from $x = 0$ to $x = 3$?

(b) How much work is done by the force in moving an object from $x = 0$ to $x = 5$?

Q3(Water-Pumping) A tank is in the shape of a downward-pointing cone (vertex at the bottom) has height 2 ft and radius 1 ft. It is filled of soda half the height of the full tank (1 ft above the bottom.) The soda weighs 63 lbs/ft^3. How much work does it take to pump all of the soda from a tank to an outlet which is at the level of the top of the tank.



$$W = \int_0^d \pi \cdot S(y) \cdot A(y) \cdot dy$$

Sec6.1. LecNote2. Sec6.2-6.4. LecNote3. Sec6.6-6.7. LecNote4.

Q4 Derivatives of the inverse functions/inverse trig/log/exp/hyperbolic functions.

(a)(Sec6.1,6.4) $f(x) = x^2 + \log_2(x+1) + 1$, find $(f^{-1})'(1)$ given $f(0) = 1$.

$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$, $(\log_a x)' = \frac{1}{\ln a} \cdot \frac{1}{x}$, $f(b) = a \Leftrightarrow b = f^{-1}(a)$

(b)(Sec6.4,6.6) $f(x) = 3^{\sin^{-1}(x)}$, find $f'(x)$ and $f'(\frac{1}{2})$.

$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$, $\int a^x dx = \frac{a^x}{\ln a} + C$

$(a^x)' = \ln a \cdot a^x$

$\sin\frac{\pi}{6} = \frac{1}{2}$

(c)(Sec6.3,6.6) $y = [\tan^{-1} x]^{\ln(\sqrt{x})}$, find y' and $y'(1)$.

$(\tan^{-1} x)' = \frac{1}{1+x^2}$

$y = f(x)^{g(x)} \Leftrightarrow \ln y = \ln f(x)^{g(x)} = g(x) \cdot \ln f(x)$.

$(\ln|x|)' = \frac{1}{x}$, $\ln x^r = r \cdot \ln x$.

$\tan\frac{\pi}{4} = 1$. $\ln 1 = 0$

(d)(Sec6.2,6.7) $y = \sinh(2x)$, find $y'(0)$ and $y''(0)$.

$\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$

$(\sinh x)' = \cosh x$, $(\cosh x)' = \sinh x$.

Sec6.5/9.3. Initial Value Problems. LecNote3.

Q5 A population $P(t)$ of insects increases according to the following law $P'(t) = k(P - 100)$. Suppose there are 500 insects at time $t = 0$, and 700 insects 5 days later. Find an expression for the number $P(t)$ of insects at time $t > 0$(in days). How many insects will there be in 5 more days?

$$\frac{dy}{dx} = \frac{f(x)}{g(y)} \iff g(y)\cdot dy = f(x)\cdot dx \iff \int g(y)\cdot dy = \int f(x)\cdot dx.$$

Q6 Find the solution to the initial value problem

$$\frac{dy}{dx} = \frac{xe^{x^2}}{y}, \quad y(0) = -3$$

Sec6.8. l'Hopital Rule. LecNote4.

Q7 Determine whether the following limits exist or not. Find the limit if it exists.

(a)

$$\lim_{x\to 0} \frac{\sec x - 1}{e^x - 1}$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

$\sec 0 = 1$, $e^0 = 1$.

$(\sec x)' = \tan^2 x$, $(e^x)' = e^x$.

(b)

$$\lim_{x\to 0} x \ln(x^2)$$

$\ln 0^+ = -\infty$.

$$0 \cdot (-\infty) = \frac{-\infty}{\frac{1}{0}} = \frac{-\infty}{\infty}$$

(c)

$$\lim_{x\to +\infty} (2x)^{\frac{1}{x}}$$

$$\infty^{\frac{1}{\infty}} = e^{\ln \infty^{\frac{1}{\infty}}} = e^{\frac{1}{\infty} \cdot \ln \infty}$$

$$= e^{\frac{\ln \infty}{\infty}}$$

Sec7.1-7.4. Methods of Integration. LecNote5. LecNote6.

Q8 Evaluate the following integrals

(a)(Sec7.1.IBP)

$$\int \underbrace{(\ln x)^2}_{u} \underbrace{\mathrm{d}x}_{dv}$$

$\int u \cdot dv = u \cdot v - \int v \cdot du$

$(\ln x)' = \frac{1}{x}$.

(b)(Sec7.2.TrigInt)

$$\int \sin^3 x \cdot \cos^{61} x \, \mathrm{d}x$$

ODD power. Sub the other one.

$(\cos x)' = -\sin x$.

$\sin^2 x = 1 - \cos^2 x$.

(c)(Sec7.3.TrigSub)

$$\int \frac{x^3}{\sqrt{x^2+1}}\mathrm{d}x$$

$$\sqrt{a^2-b^2x^2} \xrightarrow{bx=a\sin\theta} a\cdot\cos\theta$$

$$\sqrt{b^2x^2+a^2} \xrightarrow{bx=a\tan\theta} a\sec\theta$$

$$\sqrt{b^2x^2-a^2} \xrightarrow{bx=a\sec\theta} a\cdot\tan\theta$$

(d)(Sec7.4.PartialFraction.)

$$\int_0^2 \frac{10}{x^2-4x-21}\mathrm{d}x$$

$$\frac{\boxed{}}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}.$$

Sec7.8. Improper Integral. LecNote6.

Q9 Determine whether the improper integral is convergent or divergent. Evaluate the integral if it is convergent.

(a)

$$\int_0^2 \frac{1}{(x-1)^2}\mathrm{d}x = \int_0^1 \frac{1}{(x-1)^2}dx + \int_1^2 \frac{1}{(x-1)^2}dx.$$

$$\lim_{t\to 1^-}\int_0^t \frac{1}{(x-1)^2}dx \qquad \lim_{t\to 1^+}\int_t^2 \frac{1}{(x-1)^2}dx.$$

$$\left(\frac{1}{x}\right)' = \frac{-1}{x^2}$$

$$\left(\frac{1}{x-a}\right)' = \frac{-1}{(x-a)^2}$$

(b)

$$\int_4^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\mathrm{d}x = \lim_{t\to\infty}\int_4^t \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx,$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}.$$

$$e^{-\infty} = 0$$