Practice Final Version B, MTH133, Sec61, Fall 2017

Q1[6 points] Sketch the region R bounded by $y = \sqrt{1-x^2}, x = 0,\ y = x$. Set up the volume of the solid rotating R about the axis $y = 1$. Do not evaluate.

Q2[6 points] A vertical right-circular cylindrical tank measures 12 ft high and 10 ft in diameter. It is half full of kerosene weighing 20 lb/ft^3. Find the work it would take to pump the kerosene to the top of the tank.

Q3 Evaluate the following integrals.

(a)[4 points]

$$\int_0^1 xe^{2x} \, \mathrm{d}x$$

(b)[6 points]

$$\int_0^1 \frac{1}{(x)^{4/3}} \, \mathrm{d}x$$

Q4 Determine whether each of the series is convergent or divergent.

(a)[5 points]

$$\sum_{n=1}^{\infty} \frac{(2n+1)(2n-1)}{n^3}$$

(b)[5 points]

$$\sum_{n=1}^{\infty} \frac{5^n}{n!}$$

Q5[10 points] Evaluate the following integral

$$\int \frac{dx}{\sqrt{9 + x^2}}$$

Q6[6 points] Find the partial fraction decomposition for $\frac{2x^2 - x + 2}{x(x^2+2)}$ in order to evaluate

$$\int \frac{2x^2 - x + 2}{x(x^2 + 2)} dx$$

(Do not need to evaluate the integral.)

Q7[6 points]Find the equation of the line tangent to the parametric curve

$$x(t) = \arctan(2t), \quad y(t) = 3^t \qquad \text{at} \qquad (x, y) = (0, 1)$$

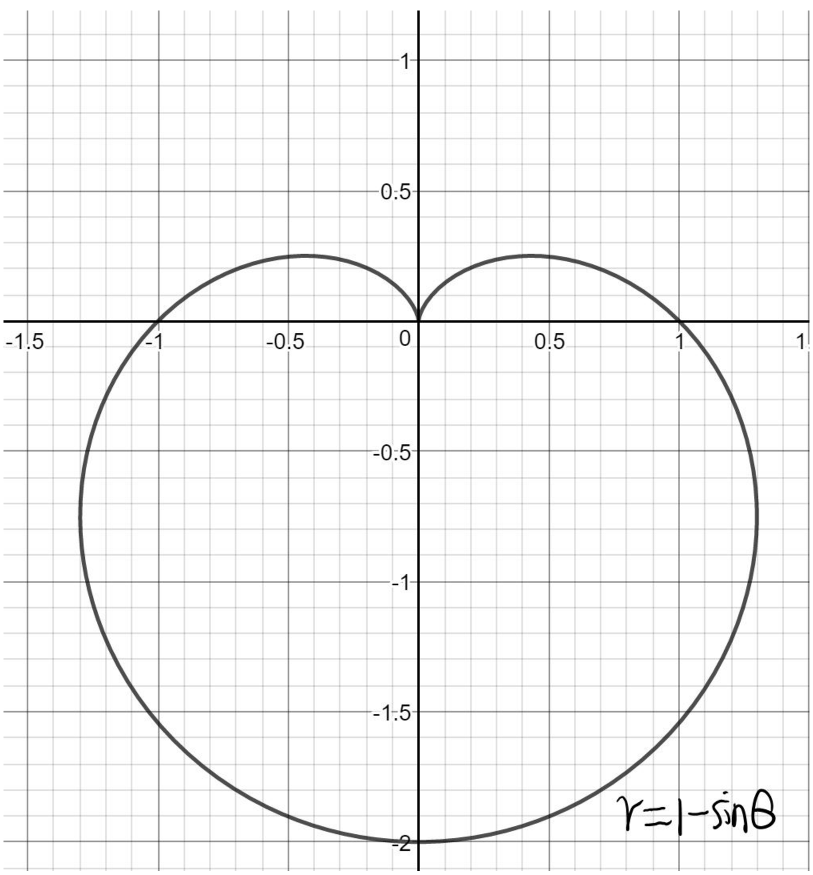
Q8[6 points] Solve $y(x)$ if

$$y'(x) = e^{-2y}x, \ y(0) = 0$$

Q9

(a) [**6 points**] Find the Cartesian equation of the polar curve given by $r = \sin\theta$. What curve is it? Sketch the curve.

(b) [**6 points**] Give the graph of Cardioid $r = 1 - \sin\theta$ as below. Find the (r, θ) coordinates of the intersection of $r = \sin\theta$ and $r = 1 - \sin\theta$ in the first quadrant. Set up the integral for the area shared by these two polar curves **in the first quadrant**. Do not evaluate.



Multiple Choice. Circle the best answer. No work needed.

Q10[3 points] Evaluate the integral

$$\int \sin\theta \cdot \sin(\cos\theta)\ \mathrm{d}\theta$$

A $\cos(\cos\theta) + C$

B $\cos(\sin\theta) + C$

C $\sin(\cos\theta) + C$

D $-\cos(\cos\theta) + C$

E $-\sin(\sin\theta) + C$

Q11[3 points] Find the open interval of convergence of the following power series.

$$\sum_{n=1}^{\infty} n(2x+1)^n$$

A $(0, 1)$

B $(0, \frac{1}{2})$

C $(-\frac{1}{2}, \frac{1}{2})$

D $(-1, 0)$

E $(-1, 1)$

Q12[3 points] Evaluate the following limit

$$\lim_{x\to 0^+} (\cos x)^{\frac{1}{x}}$$

A e

B 1

C e^2

D $-\infty$

E 0

Q13[3 points] Find the second degree Maclaurin polynomial of the function

$$f(x) = \frac{\ln(1+x)}{1-x}$$

A $x - \frac{1}{2}x^2$

B $x + \frac{1}{2}x^2$

C $1 + x + \frac{3}{2}x^2$

D $x + \frac{3}{2}x^2$

E $1 + x + x^2$

Q14[3 points] Which point given by the polar coordinates (r, θ) is in the second quadrant on the XY plane?

A $(r, \theta) = (1, \pi/4)$

B $(r, \theta) = (1, -\pi/4)$

C $(r, \theta) = (-1, 3\pi/4)$

D $(r, \theta) = (1, 3\pi/4)$

E $(r, \theta) = (1, 5\pi/4)$

Q15[3 points] Consider the sequence $a_k = \sec(\frac{2}{k})$ and the series $\sum \sec(\frac{2}{k})$

A Both the sequence and the series diverge.

B The sequence a_k converges to 1 and nth term test is inconclusive for the series.

C The sequence a_k converges to 0 and nth term test tells the series is divergent.

D The sequence a_k converges to 1 and nth term test tells the series is divergent.

E Both the sequence and the series converge.

Q16[3 points] A variable force of $\frac{6}{x^2}$ pounds moves an object along a straight line when it is x feet from the origin. Calculate the work W done in moving the object from $x = 2$ ft to $x = 3$ ft.

A 1 ft-lb

B -1 ft-lb

C 6 ft-lb

D $\frac{6}{3^2} - \frac{6}{2^2}$ ft-lb

E $\frac{6}{2^2} - \frac{6}{3^2}$ ft-lb

Q17[3 points] Which integral represents the arc-length of the parametric curve given by $x = 2\sin t, y = 3\cos t$ from $t = 0$ to $t = \pi$

A

$$\int_0^\pi \sqrt{2\sin t + 3\cos t}\ dt$$

B

$$\int_0^\pi \sqrt{2\cos t - 3\sin t}\ dt$$

C

$$\int_0^\pi \sqrt{2(\cos t)^2 + 3(\sin t)^2}\ dt$$

D

$$\int_0^\pi \sqrt{(2\sin t)^2 + (3\cos t)^2}\ dt$$

E

$$\int_0^\pi \sqrt{4(\cos t)^2 + 9(\sin t)^2}\ dt$$

Q18[3 points] Which trig-substitution can be used to evaluate

$$\int \frac{dx}{x^2\sqrt{4x^2 - 9}}$$

A $x = \tan\theta$

B $x = 4\sec^2\theta - 9$

C $x = \frac{3}{2}\sin\theta$

D $x = \frac{3}{2}\sec\theta$

E $x = \frac{2}{3}\sec\theta$

Q19[3 points] Let

$$\tan x = \sum_{n=0}^{\infty} c_n (x - \frac{\pi}{4})^n$$

be the Taylor expansion for $\tan x$ centered at $x = \frac{\pi}{4}$. Then c_1 is

A 0

B 1

C $\sqrt{2}$

D 2

E $1/\sqrt{2}$

Q20[3 points] Evaluate

$$\int_0^{\pi/4} \tan t \ \sec^2 t \ dt$$

A $\frac{1}{2}$

B 1

C $\pi/4$

D $\tan^2(\pi/4)$

E $\sec(\pi/4)$

Q21[3 points] Find the derivative of

$$\sin^{-1}(5^x)$$

A $\cos 5^x$

B $\frac{1}{\sqrt{1-5^{2x}}}$

C $\frac{5^x \ln 5}{\sqrt{1-5^{2x}}}$

D $\frac{5^x \ln 5}{\sqrt{1-5^x}}$

E $\frac{5^x}{\sqrt{1-5^{2x}}}$