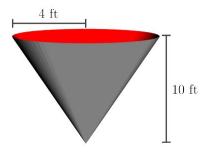
## Practice Final Version A, MTH133 Sec61, Fall 2017

 $\mathbf{Q1}[Sec 5.2, \text{ rotating solid, Washer}]$  Sketch the region R bounded by  $y=e^x, y=e, \ x=0.$ 

(a) Set up an integral for the volume of the solid rotating R about the x-axis. Do not evaluate the integral.

(b) Set up an integral for the volume of the solid rotating R about the axis y = 2e. Do not evaluate the integral.

**Q2**[Sec5.4, Water-Pumping] A conical water tank with a top diameter of 8 feet and height of 10 feet is standing at ground level as shown in the sketch below. Water weighing 60 pounds per cubic foot is pumped from the tank to the top of the tank. If the tank is full, how many foot-pounds of work are required to pump all of the water from the tank?



**Q3**[Sec6.1, derivative formula for inverse functions] Let  $f(x) = 3^x - 3x, x > 0$ . Find  $(f^{-1})'(0)$ . Notice that f(1) = 0.

Q4[Sec6.2-6.4, exp/log functions] Find the derivative of the following functions.

(a) 
$$f(x) = \ln(\sinh x)$$

(b) 
$$f(x) = \sec(\arctan x)$$

**Q5**[Sec6.5/9.3, initial value problem] Find y = y(x) if

$$\sec x \, \frac{\mathrm{d}y}{\mathrm{d}x} - \sqrt{y} = 0, \ y(0) = 4$$

 $\mathbf{Q6}[Sec 6.8/10.1,\ L'Hospital's\ Rule\ ]$  Evaluate the following limits.

(a)[
$$\infty^0$$
-type] 
$$\lim_{n\to\infty} \sqrt[n]{n^2+1}$$

(b)[
$$\infty \cdot 0$$
-type] 
$$\lim_{n \to \infty} n^2 \left( \cos \frac{1}{n} - 1 \right)$$

Q7[Sec7.1-7.2, integration by parts and trig-integral] Evaluate the following integrals.

(a) [Sec 7.1, IBP for polynomial 
$$\times \sin / \cos / \exp - type$$
] 
$$\int (x+1) \sin x \, dx$$

(b) [Sec 7.2, Odd/Even rule for sin-cos-type] 
$$\int_0^{\pi/6} (2 + \cos \theta)^2 d\theta$$

(c) [Sec 7.2, tan-sec type] 
$$\int \sec^2(2x) \tan(2x) dx$$

 $\mathbf{Q8}[Sec 7.3-7.4, \ trig\text{-}sub \ and \ partial \ fraction \ decomposition}]$  Evaluate the following integrals.

(a) [Trig-sub] 
$$\int \sqrt{4-x^2} \, dx$$

(c)[Sec7.4, P.F.D. linear product type] 
$$\int \frac{2}{t^2-1} dt$$

**Q9**[Sec 7.8, improper integral] Determine whether each of the improper integral is convergent or divergent. Evaluate the improper integral if it is convergent.

(a) 
$$\int_0^\infty \frac{2}{1 + 9x^2} \, \mathrm{d}x$$

**(b)** 
$$\int_0^{1/2} \frac{1}{\sqrt{1 - 2x}} \, \mathrm{d}x$$

 $\mathbf{Q10}[Sec11.2\text{-}11.6]$  Determine whether each of the series is convergent or divergent. Please show your work and name any test(s) that are used.

(a)[Sec11.2,n-th term test for DIV]  $\sum_{n=1}^{\infty} \cos{(\frac{1}{3n^2})}$ 

(b) [Sec11.4,(limit) Comparison Test]  $\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{n^2+1}$ 

(c)[Sec11.6, Ration Test]  $\sum_{n=1}^{\infty} \frac{n}{2^n}$ 

Q11[Sec11.6,11.8, ratio test for the radius of convergence of power series] Consider the following power series. Find its center and radius of convergence.

$$\sum_{n=1}^{\infty} \frac{3^n (x+5)^n}{n!}$$

Q12[Sec11.9/11.10, power series representation] Let

$$f(x) = \frac{2+x}{1+x^2}.$$

Find the **third degree** Maclaurin polynomial of f(x).

Q12, Taylor series Let  $f(x) = (1 + x^2) \sin x$ .

(a) Find the 2nd degree Taylor polynomial of f(x) at  $x = \frac{\pi}{2}$ .

(b) Find the first four nonzero terms in the Maclaurin series of f(x).

Q13[Sec10.1,10.2, Parametric curves/equations] Consider the parametric equations given by

$$x(t) = \ln(t+1), \quad y(t) = \sqrt{t+3}$$
 at

(a) Find the equation of the tangent line to the parametric curve at  $(x,y)=(0,\sqrt{3})$ 

(b) Set up an integral to find the arc length of the curve from t=0 to t=1 . DO NOT EVALUATE THE INTEGRAL.

## $Q14[Sec10.3,10.4,\ polar\ curve]$

(a) Sketch the curves  $r = 1 + \sin \theta$ , and r = 1. Set up the integral for the area of the region inside  $r = 1 + \sin \theta$  and outside r = 1.(Do not evaluate.)

(b) Sketch the curves  $r = 2\sin\theta$ , and  $r = 2\cos\theta$ . Set up the integral for the area of the region shared by  $r = 2\sin\theta$ , and  $r = 2\cos\theta$ . (Do not evaluate.)