READ THE FOLLOWING INSTRUCTIONS.

- Do not open your exam until told to do so.
- No calculators, cell phones or any other electronic devices can be used on this exam.
- Clear your desk of everything except pens, pencils and erasers.
- If you need scratch paper, use the back of the previous page.
- Without fully opening the exam, check that you have pages 1 through 11.
- Fill in your name, etc. on this first page.
- Show all your work. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don’t skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- There is no talking allowed during the exam.
- You will be given exactly 120 minutes for this exam.
- This is a practice exam. The actual exam may differ significantly from this practice exam because there are many varieties of problems that can test each concept.

I have read and understand the above instructions: ________________________________

SIGNATURE
1. (5 points) Let $S$ be the surface given by $x^2 + y^2 - z^2 = 1$, then
   A. $\langle 1, 1, 1 \rangle$ is tangent to $S$ at $(1, 1, 1)$.
   B. $\langle 1, 2, 1 \rangle$ is tangent to $S$ at $(1, 1, 1)$.
   C. $\langle 1, 1, -1 \rangle$ is normal to $S$ at $(1, 1, 1)$.
   D. $\langle 1, 2, 1 \rangle$ is normal to $S$ at $(1, 1, 1)$.
   E. None of the above

2. (5 points) Let $f(x, y) = xy(3 - x - y)$, then
   A. $(3, 0)$ is not a critical point of $f(x, y)$.
   B. $(3, 0)$ is a local maximum of $f(x, y)$.
   C. $(3, 0)$ is a local minimum of $f(x, y)$.
   D. $(3, 0)$ is a saddle point of $f(x, y)$.
   E. None of the above

3. (5 points) The graph of $x^2 + 2y^2 - 2z = 0$ is a(n):
   A. Ellipsoid
   B. Elliptical Paraboloid
   C. Elliptical Cone
   D. Hyperbolic Paraboloid
   E. None of the above

4. (5 points) Which of the following vector fields would match the one given in the figure:
   A. $F = \langle x, x - y \rangle$
   B. $F = \langle x, -y \rangle$
   C. $F = \langle x^2, x^2 - y \rangle$
   D. $F = \langle y, x \rangle$
True or False. No work needed. No partial credit available.

5. (2 points) For any vector \( \mathbf{u} \) and \( \mathbf{v} \), \(|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}|\). _________

6. (2 points) For any vector \( \mathbf{u} \) and \( \mathbf{v} \), \((\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = 0\). _________

7. (2 points) \((x - 2) + 3(z - 1) = 0\) is an equation for the plane passing through the point \((1, 1, 3)\) and perpendicular to \((2, 0, 1)\). _________

Extra Work Space.
8. (4 points) A vector equation of the line through the point \((0, 1, 2)\) in the direction \(v = \langle 3, 1, 0 \rangle\) can be written as ________.

9. (10 points) The domain of the function \(f(x, y) = \ln(x^2 - y + 2)\) is _________________, and range is ________________.

10. (10 points) Let \(f(x, y) = x^2 + 2 \cos(y)\), then the gradient vector field \(\nabla f\) is __________, and the rate of change of \(f\) at \((1, 0)\) in the direction of \(\langle 1, 1 \rangle\) is ________.
11. Let \( P(1, 0, 1), Q(2, 1, -3) \) and \( R(-1, 2, 0) \)

(a) (5 points) Find the projection of the vector \( \overrightarrow{PQ} \) onto the vector \( \overrightarrow{PR} \).

(b) (5 points) Compute \( \overrightarrow{PQ} \times \overrightarrow{PR} \).

(c) (5 points) Find the area of \( \triangle PQR \).

(d) (5 points) Find the distance from \((0, 0, 2)\) to the plane determined by \(P, Q,\) and \(R\).
12. (10 points) Find the length of the curve given by \( r(t) = ti + 2\cos(t)j + 2\sin(t)k \) between \( t = 1 \) and \( t = 3 \).

13. (10 points) Find \( \lim_{(x,y) \to (0,0)} \frac{3y^2 \cos^2(x)}{x^2 + 2y^2} \) if it exist, or show it does not exist.

14. (12 points) Let \( z = \sin(2x) \ln(y), \ x = t^2 - s^2, \ y = 2st \).

Use the Calc III chain rule to find \( \frac{\partial z}{\partial s} \) and \( \frac{\partial z}{\partial t} \). (No credit for other methods.)
15. (15 pts) Evaluate the double integral
\[ \int_0^1 \int_y^1 e^{x^2} \, dx \, dy \]

16. (20 points) Find the volume of the solid enclosed by \( z = x^2 + y^2 \) and \( z = 8 - x^2 - y^2 \).
17. (16 points) Find the area of the part of the surface \( z - xy = \pi \) that lies within the cylinder \( x^2 + y^2 = 16 \).

18. (12 points) Let \( \mathbf{F}(x, y) = x^2 y \mathbf{i} + (xy^2 + \frac{2}{3} x^3) \mathbf{j} \). Use Green’s theorem to find \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( C \) is the positively oriented circle \( x^2 + y^2 = 1 \).
19. Given the vector field
\[ F(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + (xy + 2z)\mathbf{k} \]

(a) (5 points) Show \( F \) is a conservative vector field.

(b) (8 points) Find a function \( f \) such that \( \nabla f = F \).

(c) (5 points) Evaluate \( \int_C F \cdot d\mathbf{r} \), where \( C \) is any smooth curve from \( P(0, 0, 1) \) to \( Q(1, 4, 1) \)
20. (16 points) For the vector field $\mathbf{F}(x, y, z) = 2\mathbf{i} + (x^2 - z^2)\mathbf{k}$, use Stokes’s theorem to evaluate

$$\int \int_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$$

where $S$ is the portion of the paraboloid $z = 4 - x^2 - y^2$, $z \geq 0$, oriented upward.

21. (16 points) Use the divergence theorem to evaluate $\int \int S \langle x^3, y^3, z^3 \rangle \cdot d\mathbf{S}$, where $S$ is the sphere $x^2 + y^2 + z^2 = 1$, oriented outward.
Congratulations you are now done with the exam! Go back and check your solutions for accuracy and clarity. Make sure your final answers are BOXED.

When you are completely happy with your work please bring your exam to the front to be handed in. Please have your MSU student ID ready so that it can be checked.

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