1. (4 points) Consider \( f(x) = \frac{x^3}{3} - x \) on the interval \([0, 3]\).
   (a) State why \( f \) satisfies the hypothesis of the MVT.

   \( f \) is a polynomial. So, it is continuous and differentiable everywhere. In particular, it is continuous on \([0, 3]\) and differentiable on \((0, 3)\).

   (b) The MVT tells us that there is a \( c \in (0, 3) \) such that \( f'(c) \) equals the slope of the secant line between \((0, f(0))\) and \((3, f(3))\). Find \( c \).

   \[
   \text{slope} = \frac{f(3) - f(0)}{3 - 0} = \frac{(9 - 3) - 0}{3} = 2
   \]

   \[
   f'(x) = x^2 - 1
   \]

   \[
   f'(c) = 2 = c^2 - 1 \implies c^2 = 3
   \]

   \[
   \implies c = \pm \sqrt{3} \quad \text{since} \quad c \in (0, 3),
   \]

   \[
   c = \sqrt{3}.
   \]

2. (3 points) Find the linearization \( L(x) \) of the function \( f(x) = \frac{x^3}{3} - x \) at the point \( x = 3 \) and use it to estimate \( f(3.1) \).

   \[
   f(3) = 6 \quad f'(3) = 9 - 1 = 8
   \]

   \[
   L(x) = 6 + 8(x - 3)
   \]

   \[
   f(3.1) \approx L(3.1) = 6 + 8(3.1 - 3) = 6 + 8(0.1) = 6.8
   \]
3. (3 points) (a) Draw a graph of a differentiable function on the interval \([-3, 3]\) that satisfies the following properties:

1. \(f(-3) = 0\)
2. \(f(3) = -3\)
3. \(f'(x) > 0\) for \(x < 0\)
4. \(f'(x) < 0\) for \(x > 0\)

(b) Where is the absolute maximum of your function from part (a)? Where is the absolute minimum?

The absolute maximum of \(f(x)\) is at \(x = 0\) and the absolute minimum is at \(x = 3\).