We consider only the set \( Z \) of integers. Let \( P \) be the statement: “The product of two even integers is even.”

(1) Write \( P \) as an implication using “if..., then...”.

Answer:

Statement \( P \) : If \( m \) and \( n \) are both even, then \( mn \) is even.

(2) Write the converse of \( P \).

Answer:

The converse of \( P \) : If \( mn \) is even, then \( m \) and \( n \) are both even.

(3) Write the inverse of \( P \).

Answer:

The inverse of \( P \) : If \( m \) or \( n \) is odd (not even), then \( mn \) is odd (not even).

(4) Write the contrapositive of \( P \).

Answer:

The contrapositive of \( P \) : If \( mn \) is odd (not even), then \( m \) or \( n \) is odd (not even).

(5) Prove that \( P \) is true.

Proof. Let \( m \) and \( n \) be both even. We show that \( mn \) is even. Since \( m \) and \( n \) are even, we write \( m = 2a \) and \( n = 2b \) for some \( a, b \in \mathbb{Z} \). Hence \( mn = (2a)(2b) = 4ab = 2(2ab) = 2c \), where \( c = 2ab \in \mathbb{Z} \). So \( mn = 2c \) is even.

(6) Prove that the converse of \( P \) is false.

Proof. Clearly, with \( m = 2 \) and \( n = 3 \), we have that the product \( mn = 2 \times 3 = 6 \) is even, but the factor \( n = 3 \) is not even. So, the converse of \( P \) is false.