(1) (10 points) Show for $n \geq 3$ that

$$u(0) = \int_{\partial B(0,r)} g(y) dS_y + \frac{1}{n(n-2)\alpha_n} \int_{B(0,r)} \left( \frac{1}{|x|^{n-2}} - \frac{1}{r^{n-2}} \right) f(x) \, dx,$$

provided $u \in C^2(\overline{B(0,r)})$ satisfies

$$\begin{cases} 
-\Delta u = f & \text{in } B(0,r), \\
 u = g & \text{on } \partial B(0,r). 
\end{cases}$$
(2) (10 points) Prove that
\[
\max_{B(0,1)} |u| \leq \max_{\partial B(0,1)} |g| + \frac{1}{2n} \max_{B(0,1)} |f|
\]
whenever \(u\) is a \(C^2(B(0,1))\)-solution of
\[
\begin{cases}
-\Delta u = f & \text{in } B(0,1), \\
u = g & \text{on } \partial B(0,1).
\end{cases}
\]
(a) (10 points) Use Poisson’s formula for ball to prove
\[ r^{n-2} \frac{r - |x|}{(r + |x|)^{n-1}} u(0) \leq u(x) \leq r^{n-2} \frac{r + |x|}{(r - |x|)^{n-1}} u(0) \]
whenever \( u \) is nonnegative and harmonic on \( B(0,r) \).

(b) (10 points) Show that a harmonic function on whole \( \mathbb{R}^n \) that is bounded above (or bounded below) must be a constant. (Compare this to the Liouville’s theorem.)
(4) (10 points) Let $u$ be the solution of

$$
\begin{cases}
\Delta u = 0 & \text{in } \mathbb{R}^n_+,

u = g & \text{on } \partial\mathbb{R}^n_+.
\end{cases}
$$

given by Poisson’s formula for the half-space. Assume $g$ is bounded and $g(x) = |x|$ for $x \in \partial\mathbb{R}^n_+, |x| \leq 1$. Show $Du$ is not bounded near $x = 0$. 