(1) Let $f_n(x) = \frac{nx}{1 + nx^2}$ for $x \in [0, \infty)$ and $n = 1, 2, 3, \cdots$.

(a) (5 points) Find the pointwise convergence limit $f(x)$ of $(f_n(x))$ for all $x \in [0, \infty)$.
(b) (5 points) Is the convergence of $(f_n) \to f$ uniform on $[0, 1]$? Justify your answer.
(c) (5 points) Show that the convergence of $(f_n) \to f$ is uniform on the set $[1, \infty)$.
(2) Let \( g_n(x) = \frac{nx^2 + 1}{x + 2n} \) for \( x \in [0, \infty) \) and \( n = 1, 2, 3, \ldots \).

(a) (3 points) Find the pointwise limit \( g(x) \) of \( (g_n(x)) \) on \([0, \infty)\).

(b) (3 points) Compute the derivative sequence \( (g_n'(x)) \).

(c) (3 points) Find the pointwise limit \( h(x) \) of \( (g_n'(x)) \) on \([0, \infty)\).

(d) (4 points) Show that the convergence of \( (g_n') \) is uniform on \([0, M]\) for every \( M > 0\).

(e) (2 points) Determine all the values of \( x \) where \( g'(x) = h(x) \).
(3) Let \( f(x) = \sum_{k=1}^{\infty} \frac{\sin(kx)}{k^3} \).

(a) (7 points) Show that \( f(x) \) is differentiable and the derivative \( f'(x) \) is continuous on \( \mathbb{R} \).

(b) (3 points) Can you determine if \( f \) is twice-differentiable? Explain briefly.