(1) Let \( f_n(x) = \frac{nx}{1 + nx^2} \) for \( x \in [0, \infty) \) and \( n = 1, 2, 3, \cdots \).

(a) (5 points) Find the pointwise convergence limit function \( f(x) \) of \( (f_n(x)) \) on \([0, \infty)\).
(b) (5 points) Is the convergence \((f_n) \to f\) uniform on \([0, 1]\)? Justify your answer.
(c) (5 points) Show that the convergence \((f_n) \to f\) is uniform on the set \([1, \infty)\).
(2) Let \( g_n(x) = \frac{nx^2 + 1}{x + 2n} \) for \( x \in [0, \infty) \) and \( n = 1, 2, 3, \ldots \).

(a) (3 points) Find the pointwise convergence limit function \( g(x) \) of \( (g_n(x)) \) on \([0, \infty)\).

(b) (3 points) Compute the derivative sequence \( (g'_n(x)) \).

(c) (4 points) Find the pointwise convergence limit function \( h(x) \) of \( (g'_n(x)) \) on \([0, \infty)\).

(d) (5 points) Show that \( (g'_n) \) converges uniformly to \( h \) on \([0, M]\) for every \( M > 0 \).
(3) Let \( f(x) = \sum_{k=1}^{\infty} \frac{\sin(kx)}{k^3} \) for \( x \in \mathbb{R} \).

(a) (7 points) Show that \( f(x) \) is well-defined and differentiable on \( \mathbb{R} \) and that the derivative \( f'(x) \) is continuous on \( \mathbb{R} \).

(b) (3 points) Can you determine if \( f \) is twice-differentiable on \( \mathbb{R} \)? Explain briefly.