(1) (8 points) Assume that \((a_n)\) is a bounded sequence with the property that every convergent subsequence of \((a_n)\) converges to the same limit \(a \in \mathbb{R}\). Show that \((a_n)\) must converge to \(a\).

(2) (8 points) Give an example of each of the following, or argue that such a request is impossible.
   (a) A sequence that is Cauchy but is not monotone.
   (b) A sequence that is monotone, but is not Cauchy.
   (c) A sequence that is unbounded and contains a subsequence that is Cauchy.
(3) (9 points) Let \((a_n)\) be a given sequence. For each \(n \in \mathbb{N}\), define \(p_n = a_n\) if \(a_n > 0\), and \(p_n = 0\) if \(a_n \leq 0\). In a similar manner, define \(q_n = 0\) if \(a_n > 0\), and \(q_n = a_n\) if \(a_n \leq 0\).

(a) Show that, if \(\sum a_n\) diverges, then at least one of \(\sum p_n\) or \(\sum q_n\) diverges.

(b) Show that if \(\sum a_n\) converges \textit{conditionally}, then \textit{both} \(\sum p_n\) and \(\sum q_n\) diverges.

(4) (9 points) Let \((a_n)\) be a sequence.

(a) Show that if the series \(\sum a_n\) converges \textbf{absolutely}, then the series \(\sum a_n^2\) converges.

(b) If \(\sum a_n\) converges, is it true that the series \(\sum a_n^2\) \textbf{must} converge? Justify your answer.

(c) If \(\sum a_n\) converges and \(a_n \geq 0\), then the series \(\sum \sqrt{a_n}\) \textit{may converge or may diverge}. Provide example to both cases.
(5) (8 points) Let $B = \left\{ \frac{(-1)^n n}{n+1} \middle| n \in \mathbb{N} \right\}$. 

(a) Find all the limit points of $B$. Make sure that there are no other limit points.
(b) Show that every point of $B$ is an isolated point of $B$.
(c) Find the closure of $B$.

(6) (6 points) Let $A \subseteq \mathbb{R}$ be a nonempty and bounded set. Show that $\sup A \in \bar{A}$.

Must $\sup A$ be a limit point of $A$? Justify your answer.
(7) (6 points) Determine whether the following statements are true or false. If the statement is true, supply a short proof, and if the statement is false, provide a counterexample.
(a) If $F_1 \supseteq F_2 \supseteq F_3 \supseteq F_4 \supseteq \cdots$ is a nested sequence of nonempty closed sets, then the intersection $\cap_{n=1}^{\infty} F_n \neq \emptyset$.
(b) An arbitrary intersection of compact sets is compact.

(8) (6 points) Determine whether the following statements are true or false. If the statement is true, supply a short proof, and if the statement is false, provide a counterexample.
(a) $\overline{A \cup B} = \overline{A} \cup \overline{B}$.
(b) $\overline{A \cap B} = \overline{A} \cap \overline{B}$. 
