1. (5 points) Prove that $\sqrt{3}$ is irrational.

2. (10 points) If a set $A$ contains $n$ elements, prove that the number of different subsets of $A$ is $2^n$. (Keep in mind that the empty set $\emptyset$ is considered to be a subset of every set.)
3. (10 points) Write a formal definition for the greatest lower bound \( \inf A \) for a set \( A \subseteq \mathbb{R} \), in the style of sup \( A \). Then use the Archimedean Property to prove that \( \inf \{ \frac{1}{n} : n \in \mathbb{N} \} = 0 \).

4. (10 points) Let \( S \) be the set consisting of all sequences of digits 0 and 1. Show that \( S \) is not countable.
5. Let $\mathbb{Q}$ be the set of rational numbers and $\mathbb{I}$ be the set of irrational numbers.

(a) (5 points) Show that if $a, b \in \mathbb{Q}$ then $ab \in \mathbb{Q}$ and $a + b \in \mathbb{Q}$.

(b) (5 points) Show that if $a \in \mathbb{Q}$ and $t \in \mathbb{I}$, then $a + t \in \mathbb{I}$ and $at \in \mathbb{I}$ as long as $a \neq 0$.

(c) (5 points) Given any two real numbers $a < b$, show that there exists a number $t \in \mathbb{I}$ such that $a < t < b$. 
