PINCHING THEOREM FOR THE VOLUME ENTROPY

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1. Introduction

For a compact Riemannian manifold (M^n, g) with $\mathrm{Ric}\,(g) \geq (n-1)$, we have the following sharp geometric inequalities for which the equality characterizes the standard sphere $(\mathbb{S}^n, g_0)$ in each case:

1. (Bishop-Gromov) the volume $\mathrm{vol}\,(M, g) \leq \mathrm{vol}\,(\mathbb{S}^n, g_0)$;
2. (Myers-Cheng) the diameter $\mathrm{diam}\,(M, g) \leq \pi$;
3. (Lichnerovicz-Obata) the first eigenvalue $\lambda_1\,(M, g) \geq n$.

For the first inequality, Colding [Co1] proved that it is stable.

Theorem 1. *(Colding) There exists a constant ε_n s.t. if a compact Riemannian manifold (M^n, g) satisfies $\mathrm{Ric}\,(g) \geq (n-1)$ and $\mathrm{vol}\,(M, g) > \mathrm{vol}\,(\mathbb{S}^n, g_0) - \varepsilon$, then M is Gromov-Hausdorff close to $\mathbb{S}^n$ in the sense that $d_{GH}\,(M, \mathbb{S}^n) \leq \alpha\,(\varepsilon) \to 0$ as $\varepsilon \to 0$.*

By Cheeger-Colding [ChC3], M is diffeomorphic to $\mathbb{S}^n$ if they are Gromov-Hausdorff close.

The other two inequalities turn out to be non-stable. In each case, one obtains a stable version if a stronger invariant is used instead. Regarding the second inequality, the diameter should be replaced by the radius which is defined to be

$$\mathrm{rad}\,(M, g) = \inf_{y \in M} \sup_{x \in X} d\,(x, y)\,.$$

Theorem 2. *(Colding [Co2]) There exists a constant ε_n s.t. if a compact Riemannian manifold (M^n, g) satisfies $\mathrm{Ric}\,(g) \geq (n-1)$ and $\mathrm{rad}\,(M, g) > \pi - \varepsilon$, then M is Gromov-Hausdorff close to $\mathbb{S}^n$.*

Regarding the third inequality, Petersen [P2] proved the following

Theorem 3. *There exists a constant ε_n s.t. if a compact Riemannian manifold (M^n, g) satisfies $\mathrm{Ric}\,(g) \geq -(n-1)$ and $\lambda_{n+1}\,(M, g) > n - \varepsilon$, then M is Gromov-Hausdorff close to $\mathbb{S}^n$.*

More recently, Aubry [A] proved that the result remains valid if λ_{n+1} is replaced by λ_n.

We are interested in compact Riemannian manifold (M^n, g) with $\mathrm{Ric}\,(g) \geq -(n-1)$. Notice that any metric can be scaled to satisfy this curvature assumption. Moreover, by the work of Lohkamp [L] that any compact manifold M^n with $n \geq 3$

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admits metrics with negative Ricci curvature. Nevertheless, there are two natural geometric inequalities which characterize hyperbolic manifolds in the equality case. First, we need to introduce two invariants. Let $\pi : \widetilde{M} \to M$ be the universal covering. Let $\lambda_0\left(\widetilde{M}\right)$ be the infimum of the L^2 spectrum of $\widetilde{M}$, i.e.

$$\lambda_0\left(\widetilde{M}\right) = \inf \frac{\int_{\widetilde{M}} |\nabla u|^2}{\int_{\widetilde{M}} u^2},$$

where the infimum is taken over all smooth functions with compact support. The volume entropy v is defined by

$$v = \lim_{r\to\infty} \frac{\ln \mathrm{vol} B_{\widetilde{M}}(x,r)}{r},$$

where $B_{\widetilde{M}}(x,r)$ is the ball of radius r centered at x in $\widetilde{M}$. It is well-known that $\lambda_0 \leq v^2/4$.

Theorem 4. *Let M^n be a compact Riemannian manifold with Ric $\geq -(n-1)$. Then $\lambda_0\left(\widetilde{M}\right) \leq (n-1)^2/4$ and equality holds iff M is hyperbolic.*

Theorem 5. *Let M^n be a compact Riemannian manifold with Ric $\geq -(n-1)$. Then the volume entropy satisfies $v \leq n-1$ and equality holds iff M is hyperbolic.*

Theorem 4 was proved by the second author [W] using the Kaimanovich entropy. Theorem 5, which implies Theorem 4 in view of the well-known fact $\lambda_0 \leq v^2/4$, was recently proved by the authors [LW]. A natural question is whether these two inequalities are stable. In fact, prior to our work [LW] Theorem 5 had been known under the additional condition that M is negatively curved as a theorem of Knieper [Kn]. In an unpublished manuscript in 2000, Courtois took up the stability question and proved the following

Theorem 6. *There exists a positive constant $\varepsilon = \varepsilon(n, D)$ s.t. if (M^n, g) is a compact Riemannian manifold of dimension n satisfying the following conditions*

- *g has **negative** sectional curvature,*
- $\mathrm{Ric}(g) \geq -(n-1)$,
- $\mathrm{diam}(M,g) \leq D$,
- *the volume entropy $v(g) \geq n-1-\varepsilon$,*

then M is diffeomorphic to a hyperbolic manifold (X, g_0). Moreover, the Gromov-Hausdorff distance $d_{GH}(M,X) \leq \alpha(\varepsilon) \to 0$ as $\varepsilon \to 0$.

His proof is based on the theory of Cheeger-Colding [ChC2] on almost rigidity.

The purpose of this paper is to present a different approach based on our previous work. We show that the method we developed in [LW] to prove the rigidity theorem can be strengthen to given the following rigidity theorem for $C^{1,\alpha}$ metrics.

Theorem 7. *Let M^n be a (smooth) compact manifold and g a $C^{1,\alpha}$ metric. Suppose that g_i is a sequence of Riemannian metrics on M s.t.*

1. $\mathrm{Ric}(g_i) \geq -(n-1)$ *for each i,*
2. *$g_i \to g$ in $C^{1,\alpha}$ norm as $i \to \infty$,*
3. *the volume entropy $v(g_i) \to n-1$ as $i \to \infty$.*

Then g is hyperbolic.

From this rigidity result, we can deduce the Theorem of Courtois in a simple way.

2. Proof of Theorem 7

We first indicate that some of the results in our previous paper [LW] are valid for a $C^{1,\alpha}$ Riemannian metric. Let M^n be a compact smooth manifold with a $C^{1,\alpha}$ Riemannian metric g. Fix a point $o \in \widetilde{M}$ and define, for $x \in \widetilde{M}$ the function $\xi_x(z)$ on $\widetilde{M}$ by:

$$\xi_x(z) \; = \; d(x,z) - d(x,o).$$

The assignment $x \mapsto \xi_x$ is continuous, one-to-one and takes values in a relatively compact set of functions for the topology of uniform convergence on compact subsets of $\widetilde{M}$. The Busemann compactification $\widehat{M}$ of $\widetilde{M}$ is the closure of $\widetilde{M}$ for that topology. The space $\widehat{M}$ is a compact separable space. The *Busemann boundary* $\partial\widehat{M} := \widehat{M} \setminus \widetilde{M}$ is made of Lipschitz continuous functions ξ on $\widetilde{M}$ such that $\xi(o) = 0$. Elements of $\partial\widehat{M}$ are called *horofunctions*. To each point $\xi \in \widehat{M}$ is associated the projection W_ξ of $\widetilde{M} \times \{\xi\}$. As a subgroup of G, the stabilizer G_ξ of the point ξ acts discretely on $\widetilde{M}$ and the space W_ξ is homeomorphic to the quotient of $\widetilde{M}$ by G_ξ. We put on each W_ξ the smooth structure and the metric inherited from $\widetilde{M}$. The manifold W_ξ and its metric vary continuously on X_M. The collection of all $W_\xi, \xi \in \widehat{M}$ form a continuous lamination $\mathcal{W}_M$ with leaves which are manifolds locally modeled on $\widetilde{M}$. In particular, it makes sense to differentiate along the leaves of the lamination and we denote $\nabla^{\mathcal{W}}$ and $\operatorname{div}^{\mathcal{W}}$ the associated gradient and divergence operators: $\nabla^{\mathcal{W}}$ acts on continuous functions which are C^1 along the leaves of $\mathcal{W}$, $\operatorname{div}^{\mathcal{W}}$ on continuous vector fields in $T\mathcal{W}$ which are of class C^1 along the leaves of $\mathcal{W}$.

On the Busemann boundary $\partial\widehat{M}$ we can construct a family of finite measures $\left\{\nu_x : x \in \widetilde{M}\right\}$ s.t.

(1) For any pair x, y, the two measures ν_x and ν_y are equivalent with the Radon-Nikodium derivative

$$\frac{d\nu_x}{d\nu_y}(\xi) = e^{-v(\xi(x)-\xi(y))};$$

(2) for any $\gamma \in \Gamma$

$$\gamma_* \nu_x = \nu_{\gamma x}.$$

Then the measure $\nu = e^{-v\xi(x)} d\nu_o(\xi)\, dx$ is G-invariant on $\widetilde{M} \times \widehat{M}$ and hence descends to a finite measure ν on X_M. By scaling we assume ν to be a probability measure. It is then proved that for all $\mathcal{W}$ vector field Y which is C^1 along the leaves and globally continuous,

$$\int \operatorname{div}^{\mathcal{W}} Y \, d\nu = v \int \left\langle Y, \nabla^{\mathcal{W}} \xi \right\rangle d\nu. \tag{2.1}$$

Since g is $C^{1,\alpha}$, the heat kernel $p_t(x,y)$ on $\widetilde{M}$ is $C^{2,\alpha}$. As in [LW], we apply the formula to the following vector field on X_M

$$Y_t(x,\xi) = \nabla (P_t \xi)(x).$$

We now cover M by finitely many open sets $\{U_i : 1 \le i \le k\}$ s.t. each U_i is so small that $\pi^{-1}(U_i)$ is the disjoint union of open sets each diffeomorphic to U_i via π. Let $\{\chi_i\}$ be a partition of unity subordinating to $\{U_i\}$. For each U_i let $\widetilde{U}_i$ be one of the components of $\pi^{-1}(U_i)$ and let $\widetilde{\chi}_i$ be the lifting of χ_i to $\widetilde{U}_i$. By the same argument we arrive at the following formula: for any $\gamma \in \Gamma$

$$\int_{\partial \widehat{M}} \sum_{i=1}^{k} \left(\int_{\gamma \widetilde{U}_i} e^{-v\xi(x)} \Delta \left(\gamma \cdot \widetilde{\chi}_i\right) dx \right) d\nu_o(\xi) = 0. \tag{2.2}$$

We now further assume that there is a sequence of smooth metrics g_i on M s.t.

- $\mathrm{Ric}(g_i) \ge -(n-1)$ for each i,
- $g_i \to g$ in $C^{1,\alpha}$ norm as $i \to \infty$.

Lemma 1. *We have $\Delta\left(e^{-(n-1)\xi}\right) \ge 0$ in the sense of distribution, i.e. for any $\chi \in C_c^{\infty}\left(\widetilde{M}\right)$ with $\chi \ge 0$*

$$\int_{\widetilde{M}} e^{-(n-1)\xi(x)} \Delta \chi(x)\, dv(x) \ge 0.$$

Proof. We will denote by Δ_i, d_i the Laplacian and distance function on $\widetilde{M}$ w.r.t. the metric g_i. By the Laplacian comparison theorem, for any $a \in \widetilde{M}$ we have

$$\Delta_i d_i(x,a) \le (n-1) \frac{\cosh(d_i(x,a))}{\sinh(d_i(x,a))}$$

in the distribution sense, i.e. for any $\chi \in C_c^{\infty}\left(\widetilde{M}\right)$ with $\chi \ge 0$

$$\int_{\widetilde{M}} d_i(x,a)\, \Delta_i \chi(x)\, dv_i(x) \le (n-1) \int_{\widetilde{M}} \frac{\cosh(d_i(x,a))}{\sinh(d_i(x,a))} \chi(x)\, dv_i(x).$$

Taking limit yields

$$\int_{\widetilde{M}} d(x,a)\, \Delta \chi(x)\, dv(x) \le (n-1) \int_{\widetilde{M}} \frac{\cosh(d(x,a))}{\sinh(d(x,a))} \chi(x)\, dv(x). \tag{2.3}$$

Let $\xi \in \partial \widehat{M}$. Then there exists a sequence $\{a_k\} \subset \widetilde{M}$ s.t. $d(o,a_k) \to \infty$ and

$$\xi(x) = \lim_{k\to\infty} d(x,a_k) - d(o,a_k).$$

here the convergence is uniform over compact sets. As a limiting form of (2.3) we obtain

$$\int_{\widetilde{M}} \xi(x)\, \Delta \chi(x)\, dv(x) \le (n-1) \int_{\widetilde{M}} \chi(x)\, dv(x).$$

Since $|\nabla \xi| = 1$ almost everywhere, we obtain as in the smooth case

$$\int_{\widetilde{M}} e^{-(n-1)\xi(x)} \Delta \chi(x)\, dv(x) \ge 0.$$

□

From now on, we assume $\lim_{i\to\infty} v(g_i) = n-1$.

Lemma 2. *We have $v(g) = n-1$.*

Proof. Since $g_i \to g$ in $C^{1,\alpha}$, for any $\varepsilon > 0$ we have for any $v \in T\widetilde{M}$

$$(1-\varepsilon)\left|v\right|_{g_i} \leq \left|v\right|_g \leq (1+\varepsilon)\left|v\right|_{g_i}$$

for i sufficiently large. It follows that

$$B_i\left(x, \frac{r}{1+\varepsilon}\right) \subset B\left(x,r\right) \subset B_i\left(x, \frac{r}{1-\varepsilon}\right),$$

where B_i denotes a geodesic ball w.r.t. g_i. Hence

$$\frac{\log \mathrm{vol} B_i\left(x, \frac{r}{1+\varepsilon}\right)}{r} \leq \frac{\log \mathrm{vol} B\left(x,r\right)}{r} \leq \frac{\log \mathrm{vol} B_i\left(x, \frac{r}{1-\varepsilon}\right)}{r}.$$

Taking limit as $r \to \infty$ yields

$$\frac{v\left(g_i\right)}{1+\varepsilon} \leq v\left(g\right) \leq \frac{v\left(g_i\right)}{1-\varepsilon}.$$

As $\lim_{i\to\infty} v\left(g_i\right) = n-1$, we have $v\left(g\right) = n-1$. □

From (2.2), in view of Lemma 1 and Lemma 2, we can now conclude as in [LW] that for ν_o-a.e. $\xi \in \partial\widehat{M}$

$$\Delta e^{-v\xi(x)} = 0$$

in the sense of distribution. By elliptic regularity, $\xi \in C^{2,\alpha}$.

We claim that $D^2\xi = g - d\xi \otimes d\xi$. To see this, first by the Bochner formula we have for any $f \in C^\infty\left(\widetilde{M}\right)$ and $\chi \in C_c^\infty\left(\widetilde{M}\right)$ with $\chi \geq 0$

$$\begin{aligned}
& \frac{1}{2}\int \left|\nabla f\right|_{g_i}^2 \Delta_i \chi dv_i \\
= & \int \left[\left|D^2 f\right|_{g_i}^2 + \left\langle \nabla f, \nabla \Delta_i f\right\rangle_{g_i} + \mathrm{Ric}_{g_i}\left(\nabla f, \nabla f\right)\right]\chi dv_i \\
\geq & \int \left[\left|D^2 f\right|_{g_i}^2 + \left\langle \nabla f, \nabla \Delta_i f\right\rangle_{g_i} - (n-1)\left|\nabla f\right|_{g_i}^2\right]\chi dv_i \\
= & \int \left[\left(\left|D^2 f\right|_{g_i}^2 - \left(\Delta_i f\right)^2 - (n-1)\left|\nabla f\right|_{g_i}^2\right)\chi - \left\langle \nabla f, \nabla \chi\right\rangle_{g_i} \Delta_i f\right] dv_i.
\end{aligned}$$

By approximation, we can take f to be ξ in this formula which in the limit, as $i \to \infty$, yields

$$\begin{aligned}
0 &= \frac{1}{2}\int \left|\nabla \xi\right|^2 \Delta \chi dv \\
&\geq \int \left[\left(\left|D^2\xi\right|^2 - \left(\Delta\xi\right)^2 - \left|\nabla\xi\right|^2\right)\chi - \left\langle \nabla\xi, \nabla\chi\right\rangle \Delta\xi\right] dv \\
&= \int \left[\left(\left|D^2\xi\right|^2 - (n-1)^2 - (n-1)\right)\chi - (n-1)\left\langle \nabla\xi, \nabla\chi\right\rangle\right] dv \\
&= \int \left[\left(\left|D^2\xi\right|^2 - (n-1)^2 - (n-1)\right)\chi + (n-1)\Delta\xi\chi\right] dv \\
&= \int \left(\left|D^2\xi\right|^2 - (n-1)\right)\chi dv,
\end{aligned}$$

i.e. $\left|D^2\xi\right|^2 \leq n-1$. On the other hand, $D^2\xi\left(\nabla\xi, \nabla\xi\right) = \frac{1}{2}\left\langle \nabla\xi, \nabla\left|\nabla\xi\right|^2\right\rangle = 0$ and $\Delta\xi = n-1$. It is then obvious that $D^2\xi = g - d\xi \otimes d\xi$.

It then follows that $\widetilde{M}$ must be isometric to $\mathbb{R}\times\Sigma$ with the metric $g = dt^2 + e^{2t}h$ ([LW, Theorem 6]). Suppose $\{u_i\}$ is a local chart on Σ. If g is smooth, then a simple calculation shows

$$e^{-4t}R_{ijkl} = e^{-4t}R^h_{ijkl} - (h_{ik}h_{jl} - h_{il}h_{jk}).$$

As g is only $C^{1,\alpha}$, this should be interpreted in the weak sense. As $\widetilde{M}$ covers the compact M, the curvature on the left hand side is bounded uniformly in t. Therefore h must be flat in the weak sense. As a result, g is Einstein in the weak sense. It is well known (cf. [P1]) that in a local **harmonic** coordiate system $\{x_i\}$ this leads to a elliptic system in the weak sense

$$\frac{1}{2}g^{kl}\frac{\partial^2 g_{ij}}{\partial x_k \partial x_l} = (n-1)\, g_{ij} + Q_{ij}\,(g, \partial g)\,,$$

where

$$\begin{aligned} Q_{ij}\,(g,\partial g) \quad &= \quad \Gamma^k_{ij}\Gamma^l_{kl} - \Gamma^l_{ik}\Gamma^k_{jl} + \frac{1}{2}\frac{\partial g^{kl}}{\partial x_k}\left(\frac{\partial g_{il}}{\partial x_j} + \frac{\partial g_{jl}}{\partial x_i}\right) \\ &\quad -\frac{1}{2}\frac{\partial g^{kl}}{\partial x_j}\frac{\partial g_{il}}{\partial x_k} - \frac{1}{2}\frac{\partial g^{kl}}{\partial x_i}\frac{\partial g_{jl}}{\partial x_k}. \end{aligned}$$

By elliptic regularity, the harmonic coordinates are $C^{2,\alpha}$ w.r.t. the original smooth structure on $\widetilde{M}$. But they are smoothly compatible with one another and therefore define a new smooth structure on $\widetilde{M}$ w.r.t. which g is smooth. Moreover ξ is also smooth and so is the decomposition $\widetilde{M} = \mathbb{R}\times\Sigma$ with $g = dt^2 + e^{2t}h$. Then we can conclude that h is flat and (Σ, h) is simply the flat $\mathbb{R}^{n-1}$. Therefore $\left(\widetilde{M}, g\right)$ is the hyperbolic space $\mathbb{H}^n$.

3. Proof Theorem 6 and Further Remarks

We first recall the following facts on negatively curved compact manifolds.

Theorem 8. *(Gromov [G1]) Suppose M^n is a closed Riemannian manifold with $-1 \le K_M < 0$. Then*

- *There exists $C_n > 0$ s.t. $\operatorname{vol}(M) \ge C_n$;*
- *For $n \ge 8$, there exist $c_n > 0$ s.t. $\operatorname{vol}(M) \ge c_n\,(1 + \mathrm{d}\,(M))$;*
- *For $4 \le n \le 7$, there exist $c_n > 0$ s.t. $\operatorname{vol}(M) \ge c_n\left(1 + \mathrm{d}^{1/3}\,(M)\right)$.*

We now prove Theorem 6. It suffices to prove that M is close to a hyperbolic manifold in the Gromov-Hausdorff sense. Suppose this is not true, then we have a sequence (M_i^n, g_i) satisfying

(1) g_i has **negative** sectional curvature,
(2) $\operatorname{Ric}(g_i) \ge -(n-1)$,
(3) $\operatorname{diam}(M_i, g_i) \le D$,
(4) the volume entropy $v\,(g_i) \to n-1$.

such that (M_i^n, g_i) is not close to any hyperbolic manifold in Gromov-Hausdorff sense.

The sectional curvature of g_i is bounded between $-(n-1)$ and 0. By Theorem 8, $\operatorname{vol}(M_i, g_i) \ge C_n > 0$. Therefore by the Cheeger finiteness theorem, we can assume that M_i are all diffeomorphic to a same manifold M passing to subsequence. By Gromov's convergence theorem [GLP, GW, P], we can find diffeomorphisms f_i such that the metrics $\widetilde{g}_i = f_i^* g_i$ converges to a metric g in $C^{1,\alpha}$ passing to a subsequence. By Theorem 7, (M, g) is a hyperbolic manifold. This is a contradiction.

Remark 1. *A natural question is if one can relax the negative curvature assumption in Theorem 6 by an arbitrary upper bound on the sectional curvature. The difficulty is to rule out collapsing. In general, collapsing can happen of course. But it seems plausible that collapsing can be ruled out under the assumption* $\mathrm{Ric}\,(g) \geq -(n-1)$ *and* $v\,(g)$ *is close to* $n-1$.

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