309 REVIEW SHEET FOR EXAM 1

1. Matrices and systems of linear equations

(1) Solve systems of linear equations $Ax = b$
   augmented matrix, elementary row operations, reduced row-echelon form,
   Gauss-Jordan reduction
(2) Fundamental theorem of homogeneous systems.
(3) Matrix arithmetic and algebra, especially matrix multiplication
(4) $Ax$ is a linear combination of the columns of $A$.
(5) Invertible matrices
(6) Elementary matrices: performing a row operation to $A$ is the same as $EA$
(7) Theorem. $A$ is nonsingular $\iff$ $Ax = 0$ has only the trivial solution $0$ $\iff$ the reduced row echelon form of $A$ is $I$.
(8) How to find $A^{-1}$?
(9) Solving $Ax = b$ by $x = A^{-1}b$

2. Determinants

(1) Determinants: definition
(2) Theorem. one can calculate determinants by cofactor expansion using any row or any column
(3) Calculate determinants by row operations
(4) $\det(AB) = \det A \det B$.
(5) $A$ is invertible iff $\det A \neq 0$.

3. Vector spaces

(1) Vector spaces: axioms, examples.
(2) Subspaces.
(3) Linear combinations and the span; how to determine if a vector is in the span?
(4) Spanning set; How to determine if a set of vectors is a spanning set?