1. VECTOR SPACES

(1) Solve systems of linear equations $Ax = b$: Gaussian elimination, elementary row operations, reduced row-echelon form.
(2) Fundamental theorem of homogeneous systems.
(3) Vector spaces: axioms, examples.
(4) Subspaces and the subspace theorem.
(5) Linear combinations and the span; how to determine if a vector is in the span?
(6) Linear dependence and linear independence. How to determine if a set is linearly independent or not?
(7) Basis. How to check if a set is a basis? The half-work theorem.
(8) Dimension. The dimensions of well known examples.
(9) The 3 big theorems: the comparison theorem, the reduction theorem and the expansion theorem
(10) How to do reduction?
(11) Coordinate vectors: how to find the coordinate vector?

2. LINEAR TRANSFORMATIONS

(1) Linear transformations $T : V \rightarrow W$: definition and examples; basis properties
(2) The nullspace $N(T)$ and the range $R(T)$; rank and nullity.
(3) The rank-nullity theorem.
(4) How to determine the nullspace and range for $L_A : \mathbb{R}^n \rightarrow \mathbb{R}^m$?
(5) The matrix $[T]_{\beta}^\alpha$ of a linear transformation: $[Tv]_{\beta} = [T]_{\alpha}^\beta [v]_{\alpha}$
(6) Linear transformation and matrix multiplication
(7) Composition and the inverse
(8) The inverse of a square matrix: how to find the inverse?
(9) The rank of a matrix; the invertibility and rank
(10) Solving $Ax = b$ by $x = A^{-1}b$
(11) When does $T : V \rightarrow W$ have an inverse?
(12) Isomorphisms and the isomorphism theorem.
(13) Change bases: $[v]_{\alpha} = Q [v]_{\alpha'} , [T]_{\alpha} = Q^{-1} [T]_{\alpha'} Q$.
(14) Determinants: definition
(15) Theorem: one can calculate determinants by cofactor expansion using any row or any column
(16) Calculate determinants by row operations
(17) $\det(AB) = \det A \det B$.
(18) Invertibility and the determinant

3. EIGENVALUES AND EIGENVECTORS

(1) eigenvalue, eigenvector, eigenspace
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(2) Characteristic polynomial
(3) How to find all the eigenvalues and eigenvectors?
(4) Similar matrices; diagonalizable matrices
(5) How to diagonalize a matrix?
(6) Application: solve systems of 1st order differential equations

4. INNER PRODUCT SPACES

(1) The dot product on $\mathbb{R}^n$
(2) Inner product: definition and examples
(3) Cauchy-Schwartz and triangle inequalities.
(4) Projection $\text{proj}_v (w)$
(5) Orthonormal set
(6) Orthogonal projection
(7) The Gram-Schmidt orthonormalization