Office hours.— My office is C300 Wells Hall. Office hours are 9:15a – 10:45a on Thursdays and 2p – 3:30p on Tuesdays, or by appointment (e-mail wongwwy@math.msu.edu). For questions about the class please also make use of our Piazza page (see §7). Questions posted on Piazza will generally receive a response within one working day; e-mails will receive responses within three working days.

Important dates.—

- Sep 7  Labour day: no class.
- Sep 9  Online open add period ends at 8pm.
- Sep 10-16 Students can go to Undergrad Office, C212 WH for mathematics enrollment changes.
- Sep 28  End of tuition-refund period.
- Oct 7  (tentative) First 1-hour test.
- Oct 21 Last day to drop with no grade reported.
- Nov 6  (tentative) Second 1-hour test.
- Nov 27  Thanksgiving: no class.
- Dec 9  (tentative) Third 1-hour test.
- Dec 18 Final exam 10a – noon.

Things on which you will be graded.— (See §10 for more details.)

- Content Projects, collaborative answers to problem sets, one-hour tests ($\times 3$), and final exam.
- Participation Attendance, in class presentation, and collaborative answers to problem sets.
- Literacy Clarity of argument during in-class presentations and discussions, as well as in written assessments (tests and exam), and ability to perform algebra/arithmetic correctly.

Pre-requisites.— (See §4 for more details.)

- MTH 320 Proof-writing for analysis on the real line, with emphasis on differential calculus.
- MTH 234 Vector calculus.

Students with disabilities.— Please first contact the RCPD (https://www.rcpd.msu.edu/) to discuss your needs, then schedule a meeting with me to arrange for accommodations.

Academic honesty.— You are encouraged to work and discuss with your peers in all aspects of the course except for the three one-hour tests and the final exam. Cheating on any form on the tests and exam (including making use of unauthorized written material or electronic devices, copying from others or making available for copy one’s work) will result in the nullification of any “tallies” (see §8) earned on the test or exam involved, and the filing of an Academic Dishonesty Report.
Contents

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§1 Welcome to MTH 421: Analysis II!

This course is a continuation of MTH 320, where you’ve learned about proof-writing in the context of single-variable differential calculus. In this class, you will further develop your skills in mathematical reasoning by practicing them on both integral and differential calculus in single- and multiple-variables. Please take the time to read through this entire document as it contains important classroom policies as well as my teaching philosophy.

Before going into details, let me start by pointing out a few things about this class that is different from the traditional lecture format:

**Hybrid lecture/discussion classes** While I will also take to the board to explain concepts, this class will involve a large amount of student participation. In particular, ideally you (collectively) will spend more time presenting your ideas at the board, in front of your peers, than I will (see §6).

**You don’t have to take the final** Your grades on the tests and the exam are determined by tallying the number of learning goals you attained (see §8 and §10, and refer to Table 1 below). I use an additive grading schema so that if you’ve maxed out on the learning goals needed, you don’t have to show up for the final exam.

**Everyone can have a perfect homework grade** I not only encourage you to collaborate on homework; I demand that you do. For a large portion of the homework assignments, the entire class will collectively produce and revise one written response (see §7.3). Throughout the term you can continue to improve the responses, and I will continue to provide feedbacks. Only the final product at the end of the term will be graded.

**You will constantly be challenged** Homework will be difficult; rather than asking you to practice what you’ve learned, most homework exercises ask you to explore what you have not yet encountered. The discovery process forms a big part of this course.

§2 My manifesto

First of all, *I am not your enemy*. My goal as a teacher is to help prepare you for success in your chosen careers (which, since you are in this class, may very well contain some mathematical aspect). The design of this class, from the learning goals to the class structure, from the method of assessment to format of homework, is for creating a low-stakes environment where you can acquire skills valuable to mathematics-related careers. To put in a skiing analogy: this entire classroom experience, everything described in this document, is a bunny hill. My role, then, is like that of the ski instructor. I will give you tips and tricks to help you stay on your feet, I will point out bad postures to help prevent your injuring yourselves, and when you fall I will help you get up and try again.

Ultimately, after taking this class, I want you to be confident in your own mathematical maturity. Back in MTH 299 you learned to think like a mathematician; after this class you will be a mathematician. So throughout the whole course I will do my best to help you develop the ability to independently consume, understand, and evaluate mathematical ideas and literature; you may have heard this ability referred to as mathematical maturity.

In this context, the actual subject of this course, “mathematical analysis”, is secondary. Going back to the skiing analogy, memorising the fine details of the theory of Riemann integration and multivariable differential calculus is much like memorising the exact layout of every bump and slope on a bunny hill. Its usefulness in the grand scheme of things is limited. Throughout this course, I
hope you can keep this perspective in mind, that you should aim to develop skills that are transferable to other situations.

With that in mind, for this class instead of the traditional “I, We, You” model of lecturing, we will do something closer to the “You, Y’all, We” model of education. Instead of my showing you directly the “establishment method” to solve problems (which, by the way, you can read in the books I listed in §11), you will, through discussions with your peers, find your own way to a solution, which we will then compare with the one given in the standard literature. Yes, this class structure is harder and require more mental concentration both for you and for me, and we may end up covering less material on paper. But the emphasis on the process of discovery and understanding is precisely what is required to train your mathematical maturity. Furthermore it is also more rewarding, since you will, on your own steam, rediscover in one semester mathematical knowledge crystallized from centuries of research.

§3 Levels of proficiency (or Bloom’s taxonomy adapted for mathematics)

There are roughly six different levels of proficiencies when it comes to mastering mathematics; I list them in Table 1 below in order of depth. The levels stack implicitly, meaning that completing, for example, the Syllogistic level of proficiency requires also completing the Nominal and Definitive levels. The last column corresponds roughly to what that level of mastery will earn you in this class.

As suggested by the table, the Inventive level of proficiency is beyond the scope of this course. The goal of this course is, in the context of mathematical analysis, to get you past the Analytic level and well into the Imitative.

§3.1 Problem-solving.— The levels presented in Table 1 form not only a tool of assessment. They can also be used to guide your problem-solving strategy. The very first step to problem solving is always

• Understand the problem: read the question carefully, identify the mathematical statements that are parts of the hypothesis, identify the mathematical statements that are parts of the desired conclusion, identify the named mathematical objects, recall other mathematical statements containing the same keywords. This uses mostly the Nominal and Comprehensive skills.

After that, one in general attacks the problem by combining two approaches.

• Reduction approach: identify known mathematical statements with similar hypotheses and conclusions to the problem, revisit their proofs and identify logical gaps caused by the changes to the hypotheses and/or conclusions, reduce the original mathematical problem to smaller steps, and re-iterate. This uses mostly the Imitative and Analytic skills.

• Growth approach: identify known mathematical statements with either the same hypotheses or conclusions as the problem, grow the chains (or trees) of implications until they meet in the middle. This uses mostly the Sylogistic skill.

These roughly correspond one-to-one with Bloom’s taxonomy, except that the Sylogistic skill is a specific aspect of what’s usually called “Application” in Bloom’s, that the Imitative and Inventive level are two different aspects of Bloom’s “Synthesis”, with a little bit of “Evaluation” mixed in.

There are other approaches, but they fall more under the Inventive level of proficiency.
Table 1: Levels of proficiency for mastering mathematics. The bullet points in the “example skills” column complete the clause “Students will demonstrate that he or she can . . .”

<table>
<thead>
<tr>
<th>Level</th>
<th>One-phrase Description</th>
<th>Example Skills</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>Know the statements</td>
<td>• recall mathematical facts, such as definitions and theorems.</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• identify mathematical facts that are useful for solving a problem, based on recognition of keywords.</td>
<td></td>
</tr>
<tr>
<td>Comprehensive</td>
<td>Understand what the statements say</td>
<td>• perform algorithmic computations based on mathematical facts.</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• classify mathematical objects according to definitions.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• determine whether a theorem can be applied based on whether the hypotheses are satisfied.</td>
<td></td>
</tr>
<tr>
<td>Syllogistic</td>
<td>Use the statements</td>
<td>• reason deductively by chaining together small numbers of mathematical statements.</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• solve problems by directly applying theorems.</td>
<td></td>
</tr>
<tr>
<td>Analytic</td>
<td>Explain the statements</td>
<td>• reproduce proofs of textbook theorems.</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• discuss the necessity of the various hypotheses in theorem statements.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• explain the relationships between similar mathematical objects and concepts.</td>
<td></td>
</tr>
<tr>
<td>Imitative</td>
<td>Generalize the statements</td>
<td>• prove “new” theorems by making appropriate modifications to the proofs of known theorems, and disprove conjectures by recognizing such modifications are impossible.</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• formulate definitions and conjectures to capture “new” mathematical concepts via analogy to known definitions and theorems.</td>
<td></td>
</tr>
<tr>
<td>Inventive</td>
<td>Discover new statements</td>
<td>• synthesize distinct mathematical concepts into something new.</td>
<td>!!!</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• formulate conjectures based on observations of special cases; prove them.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• reformulate a mathematical concept from a new point of view.</td>
<td></td>
</tr>
</tbody>
</table>
§4  Topics and Prerequisites

During this course you will explore a range of topics in mathematical analysis. In order we will discuss

1. Riemann integration of functions of one-variable;
2. structure of higher-dimensional Euclidean spaces;
3. differential calculus in multiple-variables and optimisation; and
4. (optional) implicit and inverse-function theorems, basic manifold theory;

the final subject we will touch on only if time permits and its content will not appear on the written assessments. You can find a tentative schedule of course events in Table 2 below.

You are expected to already be familiar with both

• some multivariable calculus (e.g. the contents of MTH 234) and
• proof-writing in one-variable differential calculus (e.g. the contents of MTH 320).

The in-class and online discussions will likely draw heavily from material in both. In the interest of time I may ask for questions concerning material covered in those classes to be deferred to our online discussion board.

Table 2: Course schedule, long form

<table>
<thead>
<tr>
<th>Date</th>
<th>Topic</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>9/2 W</td>
<td>—</td>
<td>Welcoming remarks</td>
</tr>
<tr>
<td>9/4 F</td>
<td>Area under the curve for piecewise linear functions</td>
<td></td>
</tr>
<tr>
<td>9/7 M</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>9/9 W</td>
<td>Darboux integrability; Riemann-Darboux integral</td>
<td>Labour day, no class</td>
</tr>
<tr>
<td>9/11 F</td>
<td>Integrability of continuous functions; integrability on subintervals</td>
<td></td>
</tr>
<tr>
<td>9/14 M</td>
<td>Error estimates for integrals of $C^{0,1}$ functions</td>
<td></td>
</tr>
<tr>
<td>9/16 W</td>
<td>Error estimates for integrals of $C^{1,1}$ functions</td>
<td>Project 1 assigned</td>
</tr>
<tr>
<td>9/18 F</td>
<td>Riemann sums and Riemann integrability</td>
<td></td>
</tr>
<tr>
<td>9/21 M</td>
<td>Lebesgue criterion</td>
<td></td>
</tr>
<tr>
<td>9/23 W</td>
<td>Properties of the integral operator: linearity, comparison principle, products of integrable functions</td>
<td></td>
</tr>
<tr>
<td>9/25 F</td>
<td>Mean value theorem</td>
<td></td>
</tr>
<tr>
<td>9/28 M</td>
<td>Mean value theorem</td>
<td>End of tuition-refund period</td>
</tr>
<tr>
<td>9/30 W</td>
<td>The primitive and its continuity</td>
<td></td>
</tr>
<tr>
<td>10/2 F</td>
<td>Fundamental theorem of calculus</td>
<td>Project 2 assigned</td>
</tr>
<tr>
<td>10/5 M</td>
<td>—</td>
<td>Extra buffer</td>
</tr>
</tbody>
</table>
### Table 2: Course schedule, long form (cont.)

<table>
<thead>
<tr>
<th>Date</th>
<th>Topic</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/7 W</td>
<td>—</td>
<td>Test 1</td>
</tr>
<tr>
<td></td>
<td><em>Higher dimensional Euclidean spaces</em></td>
<td></td>
</tr>
<tr>
<td>10/9 F</td>
<td>$\mathbb{R}^n$ as a vector space</td>
<td></td>
</tr>
<tr>
<td>10/12 M</td>
<td>The space of linear transformations</td>
<td></td>
</tr>
<tr>
<td>10/14 W</td>
<td>Distance function and open balls</td>
<td></td>
</tr>
<tr>
<td>10/16 F</td>
<td>Open sets and closed sets</td>
<td></td>
</tr>
<tr>
<td>10/19 M</td>
<td>Limit points, interior, closure</td>
<td></td>
</tr>
<tr>
<td>10/21 W</td>
<td>Continuous functions</td>
<td>Last day to drop with no grades reported</td>
</tr>
<tr>
<td>10/23 F</td>
<td>Compact sets</td>
<td></td>
</tr>
<tr>
<td>10/26 M</td>
<td>Heine-Borel theorem</td>
<td>Project 3 assigned</td>
</tr>
<tr>
<td>10/28 W</td>
<td>Topology and norm on the space of linear operators</td>
<td></td>
</tr>
<tr>
<td>10/30 F</td>
<td>Curves in Euclidean spaces, tangent vectors</td>
<td>Project 2.5 assigned</td>
</tr>
<tr>
<td>11/2 M</td>
<td>Tangent spaces of sets</td>
<td></td>
</tr>
<tr>
<td>11/4 W</td>
<td>—</td>
<td>Extra buffer</td>
</tr>
<tr>
<td>11/6 F</td>
<td>—</td>
<td>Test 2</td>
</tr>
<tr>
<td></td>
<td><em>Multivariable differential calculus</em></td>
<td></td>
</tr>
<tr>
<td>11/9 M</td>
<td>Differentiation along curves</td>
<td></td>
</tr>
<tr>
<td>11/11 W</td>
<td>Gâteaux and partial differentiation</td>
<td></td>
</tr>
<tr>
<td>11/13 F</td>
<td>Differentiation as linear approximation</td>
<td></td>
</tr>
<tr>
<td>11/16 M</td>
<td>Fréchet derivatives and the gradient</td>
<td></td>
</tr>
<tr>
<td>11/18 W</td>
<td>Continuity of derivatives, relation between Gâteaux and Fréchet</td>
<td>Project 4 assigned</td>
</tr>
<tr>
<td>11/20 F</td>
<td>Product and chain rules</td>
<td></td>
</tr>
<tr>
<td>11/23 M</td>
<td>Higher partial derivatives</td>
<td></td>
</tr>
<tr>
<td>11/25 W</td>
<td>Higher Gâteaux derivatives and vector fields</td>
<td></td>
</tr>
<tr>
<td>11/27 F</td>
<td>—</td>
<td>Thanksgiving, no class</td>
</tr>
<tr>
<td>11/30 M</td>
<td>Higher Fréchet derivatives and Taylor expansion</td>
<td></td>
</tr>
<tr>
<td>12/2 W</td>
<td>Hessian and critical points</td>
<td></td>
</tr>
<tr>
<td>12/4 F</td>
<td>Convexity</td>
<td></td>
</tr>
<tr>
<td>12/7 M</td>
<td>—</td>
<td>Extra buffer</td>
</tr>
<tr>
<td>12/9 W</td>
<td>—</td>
<td>Test 3</td>
</tr>
<tr>
<td>12/11 F</td>
<td>—</td>
<td>Last class, wrap-up</td>
</tr>
<tr>
<td>12/18 F</td>
<td>—</td>
<td>Final exam (10-noon)</td>
</tr>
</tbody>
</table>
§5 The MTH\textsubscript{421} study cycle

For each class you will be assigned a worksheet and a problem set.

1. Before each class (you are expected to spend at least 2 hours on this)
   (a) Scan through the worksheet, to get an overview of the lay of the land.
   (b) Work through the worksheet.
   (c) Ask questions (preferably by discussing it on our Piazza page; alternatively you can e-mail me or come to office hours.)
   (d) Check your answers to the worksheet. Discuss your answers with classmates.
   (e) Ask questions about the worksheet.
   (f) Read through the problem set.
   (g) Write down your ideas about the problem set.
   (h) Write down any questions about the problem set.

2. During each class
   (a) Ask questions (about the worksheet, about the problem set).
   (b) Present your ideas about the problem set.
   (c) Discuss and critique with classmates the ideas about the problem set.

3. After each class
   (a) Collaborate with your classmates on Google classroom and on Piazza to produce a definitive solution to the problem set.
   (b) Ask questions.
   (c) Read my feedback on your questions and on your write-up.
   (d) Edit the write-up to improve it.
   (e) Repeat these last three steps until you and your peers are satisfied.

Note: the study cycle has a long tail! The “after each class” step can last as long as the rest of the semester; so don’t let it hold you up from starting the study cycle for the next class!

Also, it is important to know that I will not be providing official solutions to worksheets and problem sets. This is in large part because I want you to get away from the idea that there is only one “right” way of solving problems. If you want to find out how other people answered the questions, for the worksheets you should come to class and pay attention, or ask your classmates on our Piazza page. For the problem sets you should participate in the online discussion and help write the collaborative answer.
§6 In class participation

An important skill for mathematicians (and those whose work involve the mathematical sciences) is the ability to effectively communicate and discuss topics of mathematical nature. The practice of this skill, as well as the content knowledge you will acquire and retain with the help of the active learning environment, are the main reasons for the emphasis in this course on classroom participation.

Each class will begin with taking questions on the worksheet and problem sets assigned for that day. The floor is open, in the sense that both you and I may ask questions, and both you and I may provide answers. During this time I may invite individual members of your class to share their solutions to the worksheets and problem sets. After which we will launch into the main discussion portion of the class concerning the solution to the problem sets. In this second portion the discussion will be largely student-led. I will moderate the discussion somewhat if we are veering off topic either mathematically or generally, and in rare situations I may jump in and be more hands-on to guide the discussion. In general, however, I will try to give you (almost) free-rein.

Now, some ground rules and suggestions.

• If you are presenting and you are veering off topic, I will cut you off. This is in the interest of time and for the benefit of the whole class. Please do not be offended when that happens.
• At any point of the discussion anyone can interrupt to ask questions.
• During the discussion I may interrupt and ask another student to rephrase or summarize what has just been said. This serves two functions: first is to make sure all of you are paying attention in class, the second is to make sure the presentation made by one student is at a suitable level so as to not fly over the heads of everyone else.
• In the middle of discussion I may interrupt to point out particular insightful remarks or questions; these are tied with the award of bonus points (see §10.6).
• To help speed up the presentations, I ask that you prepare a cleanly written copy of your solutions to the worksheet and problem sets and bring them to class. We will make extensive use of the document projector.
• While you should interrupt the discussion with questions or other mathematical contributions, I ask that you be respectful toward your fellow students. I ask also for your help in creating a welcoming and cordial environment. If I see behaviour that I think is antagonizing or offensive to other student(s), at the first instance I will pull you aside after class to let you know of the problem, at the second instance I will ask you to stop publicly. Thereafter I may ask you to leave the class if you are being disruptive.

§7 Online participation

Instead of D2L, we will be using Google Classroom due to its support for collaborative editing of documents (through Google Docs). Participation in Google Classroom is mandatory: this is the avenue where you will receive class announcements and assignments.

For Q+A (both of mathematics and logistics) we will be using Piazza due to its support for:

• easy integrated display of mathematics via MathJax;
• the ability for students to private message the instructor(s);
• the ability for students to post anonymously.

Participation on Piazza is optional but highly encouraged. I urge you to spend some time reading
the discussions prompted by questions posted by your peers, as well as join in and help answer the questions of your classmates.

§7.1 Getting started with Google Classroom.—

2. Sign in using your <netid>@msu.edu
   • The link in the first step should bring you to the MSU NetID sign-in page.
   • If instead you see a Google sign-in page, please enter your full MSU e-mail address (<netid>@msu.edu) as the e-mail, and leave the password field blank. Clicking the sign-in button now will bounce you over to the MSU NetID sign-in page.
3. If you have never used Google Classroom before, click the button to indicate you are a Student.
4. On the Home page, click the “+” icon.
5. Enter the code REDACTED in to the box and click “JOIN”.

§7.2 Getting started with Piazza.—

1. Visit the url REDACTED and sign-up using your MSU e-mail address.
2. After sign-up, you can visit the page at REDACTED.

§7.3 Collaborative homework.— For the collaborative homeworks which contribute to your grade, namely the 5 projects and the 34 problem sets, you, as a class, are expect to collaborate to turn in one definitive solution. This is the main reason we are using Google Classroom. A few things to note

• The collaborative responses will be made on a Google Doc that all of you have access to (once you join the course). Please be respectful of other students:
  – Please do not write anything obscene, or personally offensive in the Document.
  – Please do not include spam or otherwise deface or vandalize your classmates’ collective effort.
  – Please do not copy the contents of the Google Docs and make them available publicly elsewhere, as you and your classmates together hold both the copyright and intellectual ownership of the contents.

Note that Google Documents keep revision histories, and each person’s contribution can be tracked.
• Make sure to edit the Google Document using your own MSU NetID, as this contributes to your mathematical literacy grade.
• Please familiarize yourself with the equation editor in Google Doc. If you are familiar with the \( \text{\LaTeX} \) typesetting system or MathJax, note that the equation editor supports the input of mathematical symbols using \( \text{\LaTeX} \)-like key combos.
§8 Tests and exams

This course will have 3 one-hour-long tests and 1 two-hour-long final exam. Together they form the one major component to your final grade.

Each test will focus on one of the three major themes of the course: Riemann integration theory for functions of a single variable, structure of higher dimensional Euclidean spaces, and multivariable differential calculus. Each test will contain 3 problems, each may have multiple parts. The final exam will be comprehensive in scope and cover the material through the entire course. There will be 6 problems, 2 on each of the three themes, and each may have multiple parts. Therefore in total there will be 5 problems for each thematic focus, and 15 problems in all between the tests and the exam.

§8.1 The tally system.— For each of the 15 problems you will be judged on whether you have demonstrated the Nominal, Comprehensive, Syllogistic, and Analytic proficiencies described in Table 1; therefore for each problem you will receive a grade that is some subset of \{N, C, S, A\}.

Your goal is to obtain 3 each of the N, C, S, A marks for each of the three thematic subjects; that makes 36 marks total. Note that extra marks will not help you improve our grade; so if you have received all possible marks on the first test, you can safely skip all the Riemann integration questions on the final exam with impunity. Here I include Table 3 to help you keep track of your progress.

<table>
<thead>
<tr>
<th></th>
<th>Riemann Integration</th>
<th>Euclidean Spaces</th>
<th>Multivariable Calculus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test A</td>
<td>N C S A</td>
<td>N C S A</td>
<td>N C S A</td>
</tr>
<tr>
<td>Test B</td>
<td>N C S A</td>
<td>N C S A</td>
<td>N C S A</td>
</tr>
<tr>
<td>Test C</td>
<td>N C S A</td>
<td>N C S A</td>
<td>N C S A</td>
</tr>
<tr>
<td>Exam A</td>
<td>N C S A</td>
<td>N C S A</td>
<td>N C S A</td>
</tr>
<tr>
<td>Exam B</td>
<td>N C S A</td>
<td>N C S A</td>
<td>N C S A</td>
</tr>
<tr>
<td>Total (max 3)</td>
<td>N: C: S: A:</td>
<td>N: C: S: A:</td>
<td>N: C: S: A:</td>
</tr>
</tbody>
</table>

§8.2 Absence policy.— If your absence is unexcused, you receive zero credit for the corresponding test or exam. Excused absences for the final exam will be dealt with on a case-by-case basis. For excused absences on tests, your raw score in the thematic topic involved (used for the purpose of determining your final grade; see §10) will be computed based only on your exam performance. See next section for details.

§8.3 Converting tallies into a number.— For each of the three themes of Riemann integration, higher-dimensional Euclidean spaces, and multivariable calculus, you can earn up to 12 points toward the raw score, making a total of 36. Your total raw score from the tests and exams is made up of the sum of the three subject scores.

If you were present for the relevant test, or if you have an unexcused absence, the raw score for the corresponding theme is determined by the algorithm below, using the values of N, C, S, and A (which recall is capped to be at most 3 each) from the entries in the final row of Table 3.
1. If \( N < 3 \), then your subject raw score is \( N + \frac{1}{2}(C + S + A) \).

2. If \( N = 3 \) and \( C < 3 \), then your subject raw score is \( N + C + \frac{1}{2}(S + A) \).

3. If \( N = C = 3 \) and \( S < 3 \), then your subject raw score is \( N + C + S + \frac{1}{2}A \).

4. If \( N = C = S = 3 \), then your subject raw score is \( N + C + S + A \).

If for one or more of the tests you have an excused absence, the modified formula for the relevant subjects are:

1. If \( N = 0 \), the subject raw score is \( \frac{3}{4}(C + S + A) \).

2. If \( N \geq 1 \) and \( C = 0 \), the subject raw score is \( 3 + \frac{3}{4}(S + A) \).

3. If \( N, C \geq 1 \), and \( S < 2 \), the subject raw score is \( 6 + \frac{3}{2}S + \frac{3}{4}A \).

4. If \( N, C \geq 1 \) and \( S = 2 \) and \( A = 0 \), the subject raw score is 9.

5. If \( N, C, A \geq 1 \) and \( S = 2 \), the subject raw score is 12.

The reason for the weights is that it is important to develop lower-level proficiencies before trying the higher-level ones; one should learn to walk before learning to run. So the scoring system rewards those who preferentially work on the basics before moving on to the higher-level proficiencies.

§8.4 General instructions for tests and exams.—

- You are not allowed to receive help for any third party, including, but not limited to, copying from your classmates, seeking assistance online or via telephone or text messages.
- You are not allowed to give help to other students, including, but not limited to, allowing the copying of your work.
- All electronic devices must be turned off and stowed for the duration of test or exam, this includes mobile phones, laptops, tablet computers, calculators, and pagers. If your phone rings during the test, then for the purpose of this course you have “cheated” on the exam.
- You may not consult any printed or written material not included in the examination booklet, except for one 3-by-5 index card which summarizes Table 3.
- Once the testing material has been distributed, no students may leave the classroom in the first 15 minutes of the test (25 for exams); similarly, after the first 15 minutes of the test (25 minutes for exams) have elapsed, no students may be admitted.
- There are no bathroom breaks allowed during the 1-hour tests; you can leave the room only after turning in your paper. (In case of medical emergency you will be granted an excused absence.) During the final exam, only one student can go to the bathroom at any one time.
- Tests will be made available for review and collection two working days after the tests. You can either come pick it up during office hours or schedule a separate appointment. Once the graded tests leave my office, the mark received is final. Any challenges to the mark received must be lodged before then. If necessary you may come consult your graded tests multiple times in my office before finally collecting them.
§9 Projects

In addition to the regular assignments for every class, you will also be assigned projects approximately every two to three weeks. Like the problem sets, the written responses to projects are meant to be worked on by the entire class together; unlike the problem sets, there will be no class-time devoted to working on the projects. You are asked to collaborate with your classmates through our online platforms in Google Classroom and/or Piazza; I will monitor the discussions and provide feedback. You are free to revise your responses until the last day of classes.

Most importantly, these projects are the only places where we practice and assess the Imitative level of proficiencies described in Table 1. Since our main goal is to get you to the Analytic level, all the other assigned works (worksheets, problem sets) and written assessments (tests and exams) will only cover the first four levels of proficiencies. That a 4.0 grade in this class can only be attained if you, as a class, can work well together on the projects is deliberate: teamwork and the ability to discuss and convince each other of your ideas are two of the traits I wish you to develop during this class.

§9.1 Grading.— As you can see in Table 2, there are 5 projects in this class, Projects 1–4 and Project 2.5. Projects 1–4 will be worth 2 points each, and Project 2.5 will be worth 1 point; this makes a total of 9 points available. For each project, 25% of the grade is based on mathematical correctness of the statements, another 25% on your mathematical literacy (clarity of argument, style of writing), and the remaining 50% of the grade is based on your proficiency with the material, with 10% for each of Nominal, Comprehensive, Syllogistic, Analytic, and Imitative levels.

In addition, participation in the projects (as recorded on Piazza or through the revisions history in Google Doc) will positively impact your participation grade. So when editing the collaborative response, remember to use your own MSU log-in!

§10 Grading

The final grade will be computed from a raw score which has a baseline of 50 points: this is based on exam and tests (36), projects (9), problem sets (3.4), and a qualitative mathematical literacy score (1.6). In addition, there are three modifiers: in-class presentation, insight bonus, and attendance. See below for detailed description.

§10.1 Exam and tests.— Together they contribute up to 36 points to the raw score. See §8 for more details.

§10.2 Projects.— Together they contribute up to 9 points to the raw score. See §9 for more details.

§10.3 Problem sets.— There are a total of 34 problem sets, each is worth 0.1 point toward the raw score. The number of points awarded is all-or-nothing for each problem set, since I will be providing feedback throughout the semester. Altogether this makes available 3.4 points.

§10.4 Mathematical literacy.— You will be graded on your mathematical literacy, meaning your ability to give clear mathematical arguments, write in good mathematical style, and perform basic...
algebraic and arithmetic manipulations. This counts for 1.6 points of your raw score. Note that

1. The score is qualitative; because good mathematical writing, like any good writing, is a craft.

2. The score is not cumulative: even if you start out stuttering a great deal in your mathematical reasoning, if you do well by the end of the term, you can easily earn full credit.

3. At any point during the term you can check with me for your current performance. I will also help point out, if you wish, what you need to do to improve them.

In addition to your responses on tests and the exam, I also admit into consideration your in-class discussion contributions as well as your contributions to Piazza (if any) in determining this portion of your grade.

§10.5 In-class presentation.— Each student is expected, during the term, to present substantial mathematical arguments at the board in front of his or her peers; each student is expected to participate at least twice. Students who don’t participate can earn a maximum of 2.0 in the course; students who participate only once can earn a maximum of 3.0 in the course. You can check with me at any point during the term to see if you have satisfied this requirement.

§10.6 Insight bonus.— Students will be rewarded for insightful questions, answers, or comments in class or online. Students will be informed on the spot when they received such a bonus. For example, during class I may interrupt a presentation to point out particular insightful statements; on Piazza, if I spot a particularly inspired edit or question, I will send you a note indicating as such. (Note: this is not the same as Piazza’s “instructor endorsement” function; though frequently the two will happen simultaneously.)

Each such instance adds 0.2 points toward your raw score. The total modifier resulting from these bonuses and from attendance will be capped at ±5 points total.

§10.7 Attendance and tardiness.— Since we devote a lot of time to in-class discussion, students are expected to come to class. To “encourage” that, attendance is the only place where poor showing will negatively affect your grade.

• For each excused absence beyond the first three, your raw score will be deducted by 0.1 points.
• For each unexcused absence beyond the first one, your raw score will be deducted by 0.3 points.

Please see Appendix A for what constitutes excused absences. Please note that if you are late to class by more than 20 minutes, it is counted as an absence.

The total modifier resulting from absences and the bonuses discussed in the previous section is capped at ±5 points. In essence, by making good contributions and earning bonuses, you can do “make up work” for the missing participation time due to your absence.

As a remark: we will operate on the “10 minute rule” for me. If for some reason outside my control that I am delayed by more than 10 minutes to the class, you can consider that session cancelled. The course schedule up to the next available “Extra buffer” day will be pushed back by one day.

§10.8 Determining your grade.—
Table 4: Raw score to final grade conversion chart

<table>
<thead>
<tr>
<th>Raw score</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>[48,55]</td>
<td>4.0</td>
</tr>
<tr>
<td>[41,48)</td>
<td>3.5</td>
</tr>
<tr>
<td>[34,41)</td>
<td>3.0</td>
</tr>
<tr>
<td>[27,34)</td>
<td>2.5</td>
</tr>
<tr>
<td>[20,27)</td>
<td>2.0</td>
</tr>
<tr>
<td>[15,20)</td>
<td>1.5</td>
</tr>
<tr>
<td>[10,15)</td>
<td>1.0</td>
</tr>
<tr>
<td>[−5,10)</td>
<td>0.0</td>
</tr>
</tbody>
</table>

1. First compute the modifier based on your attendance history and on the “insight bonus”; cap it appropriately at ±5 points.

2. Next compute the base score by summing the exam/test, project, problem set, and literacy scores.

3. Sum the base score and the modifier obtained, and then refer to Table 4 to find your corresponding grade.

4. Apply any caps resulting from giving fewer than 2 in-class presentations.

§11 Additional resources

While I require no textbooks for this course, the following books can be good references. The books are listed below in no particular order, and they vary in terms of style of presentation and content covered, you should feel free to choose one that is most useful to you if needed. (N.B. Traditionally this course has been taught with Wade’s textbook as the reference.)


With the exception of the last two items (the books by Ross and by Marsden and Hoffman), copies of the above have been put on reserve in the Math Library.
§A  Excused absences

First, I remind you to refer to the University guidelines on the Ombudsperson’s website. Absences are by default unexcused; please contact me as early as possible regarding any anticipated absences.

For final exams, the only satisfactory explanations for your absence are

- You voluntarily decide not to take the final exam; likely due to the fact you have already earned the requisite tallies under our grading system; see §B.
- Per university final exam policy, you are “unable to take [the] final examination because of illness or other reasons over which [you] have no control”.

In the latter case I urge you to notify the associate dean of your college immediately, and be prepared to document your absence with supporting evidence.

Absences from the one-hour tests and normal classes are excused if

- You were ill, or you face emergencies (such as death or illness of family member, court appearances, or other circumstances which take attendance out of your control). In these situations please notify me immediately either directly or through your academic advisor. Once you return to campus and are available, please either visit my office hours or schedule an appointment to show me any documentation to justify your absence.
- You need to observe a religious holiday; participate in an university-sanctioned field trip, rehearsal, or performance; or participate in an university-approved athletic competition. Please notify me at least one week in advance of your scheduled absence and bring me supporting documentation either during my office hours or during a separate appointment.

https://www.msu.edu/unit/ombud/classroom-policies/index.html