HW 8

[Hungerford] Section 4.5, #5
Use Eisenstein's Criterion to show that each polynomial is irreducible in $\mathbb{Q}[x]$:

(a) $x^5 - 4x + 22$

Solution. Use $p = 2$, since 2 divides 4 and 22, but 2 does not divide 1, and 2^2 does not divide 22.

(b) $10 - 15x + 25x^2 - 7x^4$

Solution. Use $p = 5$, since 5 divides $10, 15$, and 25, but 5 does not divide 7, and 5^2 does not divide 10.

(c) $5x^{11} - 6x^4 + 12x^3 + 36x - 6$

Solution. Use $p = 3$, since 3 divides $6, 12$, and 36, but 3 does not divide 5, and 3^2 does not divide 6. Alternatively, you could use $p = 2$, since 2 divides $6, 12$, and 36, but 2 does not divide 5 and 2^2 does not divide 6.

[Hungerford] Section 4.5, #11 Prove that $30x^n - 91$ has not roots in $\mathbb{Q}$ (where $n > 1$).

Solution. We can factor 91 into $91 = 7 \cdot 13$. Since 30 is not divisible by either 7 or 13, and since 91 is not divisible by either 7^2 or 13^2, we can use **Eisenstein's Criterion** (with either $p = 7$ or $p = 13$) to argue that $30x^n - 91$ is irreducible in $\mathbb{Q}[x]$. Now, by **Corollary 4.18**, $30x^n - 91$ has no roots in $\mathbb{Q}$.

[Hungerford] Section 4.5, #13 Prove that $f(x) = x^4 + 4x + 1$ is irreducible in $\mathbb{Q}[x]$ by using **Eisenstein's Criterion** to show that $f(x + 1)$ is irreducible and applying **Exercise 12**.

Solution. Substituting $x + 1$ for x, we get that

$$f(x+1) = (x+1)^4 + 4(x+1) + 1 = x^4 + 4x^3 + 6x^2 + 8x + 6$$

Notice that all the coefficients but the leading one are even, so 2 divides them all. Furthermore, 2 does not divide the leading coefficient (since it is 1), and 2^2 does not divide the constant term (which is 6). So by **Eisenstein's Criterion**, $f(x + 1)$ is irreducible in $\mathbb{Q}[x]$. By the previous exercise (number 12), this means that f is also irreducible.