[Hungerford] Section 4.2, #2 If $f \in F[x]$ (for a field F), with $f = \sum_{i=0}^{n} c_i x^i$, and $c_n \neq 0$, what is the gcd of f and 0?

Solution. Since *anything* is a divisor of 0, f itself is a common divisor of f and 0. But by definition, the gcd must be monic, so we just take the monic multiple of f, given by

$$\frac{1}{c_n} f = \sum_{i=0}^{n} \frac{c_i}{c_n} x^i$$

[Hungerford] Section 4.2, #10 Find the gcd of $x + a + b$ and $x^3 - 3abx + a^3 + b^3$ in $\mathbb{Q}[x]$.

Solution. For any a and b, the linear polynomial $x + a + b$ actually divides $x^3 - 3abx + a^3 + b^3$. Since $x + a + b$ is already monic, it is in fact the gcd.

[Hungerford] Section 4.2, #13 Prove that if F is a field, and $a, b, c \in F[x]$ such that $a \mid bc$, and a coprime to b, then $a \mid c$.

Solution. Since a and b are coprime, then by **Theorem 4.8** in the book, there are polynomials $u, v \in F[x]$ so that

$$au + bv = 1$$

Multiply both sides by the polynomial c to get

$$auc + bvc = c$$

Everything commutes, so we can re-write this as

$$a(uc) + (bc)v = c$$

We see that a obviously divides the first term, and since we assume that a divides bc, then a divides the second term as well. So a divides the entire left-hand side, and so it divides c.