Section 1.1, #2. Use truth tables to show that

(a) $\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$

(b) $((P \vee Q) \vee R) \Leftrightarrow (P \vee (Q \vee R))$

(c) $((P \wedge Q) \vee R) \Leftrightarrow ((P \vee R) \wedge (Q \vee R))$

(d) $((P \vee Q) \wedge R) \Leftrightarrow ((P \wedge R) \vee (Q \wedge R))$

In the following tables, "0" represents "**F**" or "*false*", and "1" represents "**T**" or "*true*".

(a) The columns in yellow are the end result of evaluating each side of the "$\Leftrightarrow$":

P	Q	$\neg$	$(P$	$\Rightarrow$	$Q)$	$\Leftrightarrow$	$(P$	$\wedge$	$\neg Q)$
0	0	0	0	1	0		0	0	1
0	1	0	0	1	1		0	0	0
1	0	1	1	0	0		1	1	1
1	1	0	1	1	1		1	0	0

(b)

P	Q	R	$((P$	$\vee$	$Q)$	$\vee$	$R)$	$\Leftrightarrow$	$(P$	$\vee$	$(Q$	$\vee$	$R))$
0	0	0	0	0	0	0	0		0	0	0	0	0
0	0	1	0	0	0	1	1		0	1	0	1	1
0	1	0	0	1	1	1	0		0	1	1	1	0
0	1	1	0	1	1	1	1		0	1	1	1	1
1	0	0	1	1	0	1	0		1	1	0	0	0
1	0	1	1	1	0	1	1		1	1	0	1	1
1	1	0	1	1	1	1	0		1	1	1	1	0
1	1	1	1	1	1	1	1		1	1	1	1	1

(c)

P	Q	R	$((P$	$\wedge$	$Q)$	$\vee$	$R)$	$\Leftrightarrow$	$((P$	$\vee$	$R)$	$\wedge$	$(Q$	$\vee$	$R))$
0	0	0	0	0	0	0	0		0	0	0	0	0	0	0
0	0	1	0	0	0	1	1		0	1	1	1	0	1	1
0	1	0	0	0	1	0	0		0	0	0	0	1	1	0
0	1	1	0	0	1	1	1		0	1	1	1	1	1	1
1	0	0	1	0	0	0	0		1	1	0	0	0	0	0
1	0	1	1	0	0	1	1		1	1	1	1	0	1	1
1	1	0	1	1	1	1	0		1	1	0	1	1	1	0
1	1	1	1	1	1	1	1		1	1	1	1	1	1	1

(d)

P	Q	R	$((P$	$\vee$	$Q)$	$\wedge$	$R)$	$\Leftrightarrow$	$((P$	$\wedge$	$R)$	$\vee$	$(Q$	$\wedge$	$R))$
0	0	0	0	0	0	0	0		0	0	0	0	0	0	0
0	0	1	0	0	0	0	1		0	0	1	0	0	0	1
0	1	0	0	1	1	0	0		0	0	0	0	1	0	0
0	1	1	0	1	1	1	1		0	0	1	1	1	1	1
1	0	0	1	1	0	0	0		1	0	0	0	0	0	0
1	0	1	1	1	0	1	1		1	1	1	1	0	0	1
1	1	0	1	1	1	0	0		1	0	0	0	1	0	0
1	1	1	1	1	1	1	1		1	1	1	1	1	1	1

Section 1.3, #4: Prove that the following functions are surjective:

(a) The function $f\colon \mathbb{R} \to \mathbb{R}$ is given by $f(x) = x^3$. Just notice that for any $y \in \mathbb{R}$, we also have $\sqrt[3]{y} \in \mathbb{R}$, with the property that $f\left(\sqrt[3]{y}\right) = \left(\sqrt[3]{y}\right)^3 = y$, and so f is surjective onto $\mathbb{R}$.

(b) The function $f\colon \mathbb{Z} \to \mathbb{Z}$ is given by $f(x) = x - 4$. Notice that for any $n \in \mathbb{Z}$, $n + 4$ is also in $\mathbb{Z}$, with the property that $f(n+4) = (n+4) - 4 = n$, so f is surjective onto $\mathbb{Z}$.

(c) The function $f\colon \mathbb{R} \to \mathbb{R}$ is given by $f(x) = 5 - 3x$. Notice that for any $y \in \mathbb{R}$, $\frac{5-y}{3}$ is also in $\mathbb{R}$, with the property that

$$f\left(\frac{5-y}{3}\right) = 5 - 3\left(\frac{5-y}{3}\right) = 5 - (5-y) = y,$$

so f is surjective onto $\mathbb{R}$.

(d) The function $f\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Q}$ is given by

$$f(a,b) = \begin{cases} \frac{a}{b} & b \neq 0 \\ 0 & b = 0 \end{cases}$$

Let $q \in \mathbb{Q}$. Any rational number can be written as $\frac{a}{b}$ for integers a, b, where $b \neq 0$, so choose such a and b so that $q = \frac{a}{b}$. Then $f(a,b) = q$, and so f is surjective onto $\mathbb{Q}$.

Section 1.4, #6. Let x be a real number greater than -1. Prove that for every positive integer n, $(1+x)^n \geq 1+nx$.

We will prove this by induction. First, we'll do the base case $n = 1$. In this case we actually have equality, since

$$(1+x)^n = (1+x)^1 = 1+x = 1+1\cdot x = 1+nx$$

So in this case, we have that $(1+x)^n \geq 1+nx$. Now, supposing that $(1+x)^n \geq 1+nx$ for some positive integer n, we need to show that $(1+x)^{n+1} \geq 1+(n+1)x$. So we will factor $(1+x)^{n+1}$ and use the induction assumption:

$$\begin{aligned}
(1+x)^{n+1} &= (1+x)^n(1+x) \\
&\geq (1+nx)(1+x) && \text{(by induction)} \\
&= 1+(n+1)x+nx^2 \\
&\geq 1+(n+1)x && \text{(since } nx^2 \geq 0\text{)}
\end{aligned}$$

We have showed the base case, and the inductive step, so by induction, $(1+x)^n \geq 1+nx$ for all $n \geq 1$.