

MTH310 QUIZ 1 SOLUTIONS

1. Use a truth table to verify the statements:

I. $\neg(p \text{ or } q) \iff \neg p \text{ and } \neg q$

p	q	p or q	$\neg(p \text{ or } q)$	$\neg p$	$\neg q$	$(\neg p \text{ and } \neg q)$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

II. $(p \implies q \text{ and } q \implies r) \implies (p \implies r)$

p	q	r	$p \implies q$	$q \implies r$	$p \implies q \text{ and } q \implies r$	$p \implies r$	$(p \implies q \text{ and } q \implies r) \implies (p \implies r)$
T	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T

2. Prove that 64 is a factor of $9^n - 8n - 1$ for every nonnegative integer n.

Proof:

(1) 64 is a factor of $9^n - 8n - 1$ for every nonnegative integer n.

Let $n=0$, $9^n - 8n - 1 = 1 - 0 - 1 = 0 = 0 \cdot 64$, then (1) is true at $n=0$

Assume (1) is true at $n=k$, then we have $9^k - 8k - 1 = 64m$ ($m \in \mathbb{Z}$)

for $n=k+1$, $9^{k+1} - 8(k+1) - 1 = 9^k \cdot 9 - 8k - 9 = 9^k(9 - 1) - 8k$

$$=9*(64m+8k)-8k=9*64m+64k=64*(9m+k)$$

Since $m \in \mathbb{Z}$, $k \in \mathbb{N}$, we have $9m + k \in \mathbb{Z}$,

(1) is true at $n=k+1$

Thus, 64 is a factor of $9^n - 8n - 1$ for every nonnegative integer n .