MTH310 QUIZ 3 SOLUTIONS

Problem 1

(1) What is the definition of a field?

A field is a commutative ring R with identity $1_R \neq 0_R$ that satisfies this axiom:

For each $a \neq 0_R$ in R, the equation ax= 1_R has a solution in R.

(2) What is the definition of an integral domain?

An integral domain is a commutative ring R with identity $1_R \neq 0_R$ that satisfies this axiom:

Whenever a,b $\in$ R and ab $= 0_R$, then $a = 0_R$ or $b = 0_R$

(3) Is an integral domain always a field? If yes, prove it. If no, give a counter example.

The ring $\mathbb{Z}$ is an integral domain but not a field.

Problem 2

(1) What is the definition of zero divisors?

An element a in a ring R is a zero divisor provided that

i. $a \neq 0_R$

ii. There exists a nonzero element c in R such that ac= 0_R or ca= 0_R.

(2) How many elements in $\mathbb{Z}_{18}$ are zero divisors? Explicitly list them.

11 elements are zero divisors. They are $\{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16\}$.