MTH310 QUIZ 3 SOLUTIONS

Problem 1

(1) What is the definition of a field?

A field is a commutative ring \( R \) with identity \( 1_R \neq 0_R \) that satisfies this axiom:

For each \( a \neq 0_R \) in \( R \), the equation \( ax = 1_R \) has a solution in \( R \).

(2) What is the definition of an integral domain?

An integral domain is a commutative ring \( R \) with identity \( 1_R \neq 0_R \) that satisfies this axiom:

Whenever \( a,b \in R \) and \( ab = 0_R \), then \( a = 0_R \) or \( b = 0_R \)

(3) Is an integral domain always a field? If yes, prove it. If no, give a counter example.

The ring \( \mathbb{Z} \) is an integral domain but not a field.

Problem 2

(1) What is the definition of zero divisors?

An element \( a \) in a ring \( R \) is a zero divisor provided that

i. \( a \neq 0_R \)

ii. There exists a nonzero element \( c \) in \( R \) such that \( ac = 0_R \) or \( ca = 0_R \).

(2) How many elements in \( \mathbb{Z}_{18} \) are zero divisors? Explicitly list them.

11 elements are zero divisors. They are \( \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16\} \).