MTH310 QUIZ 2 SOLUTIONS

Problem 1

(1) Write down 8 axioms of rings. (3 points)

(A x 1) a+b $\in$ R for a, b $\in$ R (closure of addition)

(A x 2) a+(b+c) = (a+b) + c for all a,b,c $\in$ R (associative addition)

(A x 3) a+b = b+a for all a,b $\in$ R (commutative addition)

(A x 4) There exists an element in R, denote by 0_R and called "zero R" such that a= a+ 0_R and a=0_R + a for all a$\in$ R (additive identity)

(A x 5) for each a $\in$ R, there exists an element in R, denote by -a and called"negative a ", such that a+(-a) = 0_R (additive inverse)

(A x 6) ab $\in$ R, for all a, b $\in$R (closure for multiplication)

(A x 7) a(bc)=(ab)c for all a,b,c $\in$ R (associative multiplication)

(A x 8) a(b+c) = ab + ac and (a+b)c = ac + bc for all a,b,c $\in$ R (distributive laws)

(2) Check 8 axioms on the set $a + b\sqrt{2}|a, b \in \mathbb{Z}$ carefully and determine that it is a ring or not. (2 points)

Assume S= $a + b\sqrt{2}|a, b \in \mathbb{Z}$,

$a_1 + b_1\sqrt{2}$, $a_2 + b_2\sqrt{2} \in$ S,

I. $(a_1 + b_1\sqrt{2})$ + $(a_2 + b_2\sqrt{2})$ = $(a_1 + a_2)$ + $(b_1 + b_2)\sqrt{2} \in$ S

II. $(a_1 + b_1\sqrt{2})$ * $(a_2 + b_2\sqrt{2})$ = $(a_1a_2 + 2b_1b_2)$ + $(a_1b_2 + a_2b_1)\sqrt{2} \in$ S

III. 0= 0+ 0*$\sqrt{2} \in$ S

IV. The additive inverse $(-a_1) + (-b_1)\sqrt{2} \in$ S

Thus, S is a subring of $\mathbb{R}$

Therefore, S is a ring.

Problem 2

How many elements in $\mathbb{Z}_{12}$ are units? Explicitly write them down and find each one corresponding multiplication inverse.

4 elements are units. They are 1,5,7,11

The corresponding multiplication inverse of 1 is 1.

The corresponding multiplication inverse of 5 is 5.

The corresponding multiplication inverse of 7 is 7.

The corresponding multiplication inverse of 11 is 11.