Name: ______________________________

Section: ____________ **Recitation Instructor**: ______________________

READ THE FOLLOWING INSTRUCTIONS.

- **Do not open your exam until told to do so.**
- No calculators, cell phones or any other electronic devices can be used on this exam.
- Clear your desk of everything excepts pens, pencils and erasers.
- If you need scratch paper, use the back of the previous page.
- Without fully opening the exam, check that you have pages 1 through 10.
- Fill in your name, etc. on this first page.
- **Show all your work.** Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand and a proctor will come to you.
- There is no talking allowed during the exam.
- You will be given exactly 90 minutes for this exam.

I have read and understand the above instructions: ______________________________

SIGNATURE

Multiple Choice. Circle the best answer. No work needed. No partial credit available.

1. (5 points) A variable force of $x^2 - 2x$ pounds moves an object along a straight line when it is x feet from the origin. Calculate the work W done in moving the object from $x = 2$ to $x = 3$ feet.

   A. $\frac{4}{3}$ **ft-lbs**

   B. $\frac{-20}{3}$ ft-lbs

   C. 14 ft-lbs

   D. 6 ft-lbs

   E. $\frac{8}{3}$ ft-lbs

2. (5 points) Evaluate $\int \sin^4(x)\cos^3(x)\ dx$

   A. $\frac{1}{5}\sin^5(x) - \frac{1}{7}\sin^7(x) + C$

   B. $-\frac{1}{5}\sin^5(x) + \frac{1}{7}\sin^7(x) + C$

   C. $-\frac{1}{5}\cos^5(x) + \frac{1}{7}\cos^7(x) + C$

   D. $\frac{1}{20}\cos^5(x)\sin^4(x) + C$

   E. $-\frac{1}{20}\cos^5(x)\sin^4(x) + C$

3. (5 points) Let $f(x) = 2x^4 + 3x - 5$ for $x > 0$. Find $\left(f^{-1}\right)'(x)$ at the point $x = 0 = f(1)$

   A. $\frac{1}{3}$

   B. 11

   C. -5

   D. $\frac{1}{11}$

   E. undefined

4. (5 points) Find $f'(x)$ if $f(x) = \cos^{-1}(3x)$.

   A. $f'(x) = \dfrac{1}{\sqrt{1-9x^2}}$

   B. $f'(x) = \dfrac{3}{\sqrt{1-9x^2}}$

   C. $f'(x) = 3 \cdot \cos^{-2}(3x) \cdot \sin(3x)$

   D. $f'(x) = \dfrac{3}{1+9x^2}$

   E. $f'(x) = -\dfrac{3}{\sqrt{1-9x^2}}$

5. (5 points) For the integral $\int \sqrt{x^2-49}\, dx$, with $|x| \le 7$, the following trigonometric substitution should be made.

   A. $x = 7\cos t$

   B. $x = 7\sin t$

   C. $x = 7\sec t$

   D. $x = \sin t$

   E. $x = 7\tan t$

6. (5 points) Find the partial fraction decomposition of $\dfrac{5x+2}{x^2+x}$.

   A. $\dfrac{3}{x} + \dfrac{2}{x+1}$

   B. $\dfrac{5x}{x} + \dfrac{2}{x+1}$

   C. $\dfrac{2}{x} + \dfrac{5x}{x+1}$

   D. $\dfrac{2}{x} + \dfrac{3}{x+1}$

   E. $\dfrac{5x}{x^2} + \dfrac{2}{x}$

7. (5 points) Compute $\displaystyle\int_0^1 \frac{x}{(x+1)^2} dx$.

   A. $\ln 2 - 1$

   B. $\ln 2$

   C. $\ln 2 + \dfrac{1}{2}$

   D. $\ln 2 - \dfrac{1}{2}$

   E. $1/6$

8. (5 points) Find $f'(x)$ if $f(x) = \log_2 2^x$.

    A. $f'(x) = \dfrac{\ln 2^x}{\ln 2}$

    B. $f'(x) = 1$

    C. $f'(x) = x \cdot \log_2 2$

    D. $f'(x) = \dfrac{1}{2^x}$

    E. $f'(x) = 0$

9. (5 points) Find the equation of line tangent to the curve $f(x) = x^{\cos x}$ at the point $(\pi/2, 1)$.

    A. $y - 1 = 1(x - \pi/2)$

    B. $y - 1 = -\ln(\pi/2)(x - \pi/2)$

    C. $y - 1 = \ln(\pi/2)(x - \pi/2)$

    D. $y = 1$

    E. $y - 1 = \ln(\pi/2 + 1)(x - \pi/2)$

10. (5 points) Evaluate $\displaystyle\int \frac{\sinh\sqrt{x}}{\sqrt{x}} dx$

    A. $\frac{1}{2}\cosh\sqrt{x} + C$

    B. $-\frac{1}{2}\cosh\sqrt{x} + C$

    C. $2\cosh\sqrt{x} + C$

    D. $-2\cosh\sqrt{x} + C$

    E. None of the above.

Standard Response Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

11. (16 points) Evaluate the following limits.

(a) $\displaystyle\lim_{x\to 0}\frac{e^x-1}{x^2+3x}$

Solution: $\dfrac{0}{0}$ case

L'Hop

$\displaystyle = \lim_{x\to 0}\frac{e^x}{2x+3}=\frac{1}{3}$

(b) $\displaystyle\lim_{x\to 0^+} x\big(\ln(2x)\big)$

Solution: $0\cdot -\infty$ case

Rewrite $\displaystyle =\lim_{x\to 0^+}\frac{\ln 2x}{x^{-1}}$ $\quad\dfrac{\infty}{\infty}$ case

L'Hop

$\displaystyle = \lim_{x\to 0^+}\frac{x^{-1}}{-x^{-2}}=\lim_{x\to 0^+}-x=0$

12. (24 points) Evaluate the following integrals.

(a) $\displaystyle\int xe^{2x}\ dx$

Solution:

$u = x \quad dv = e^{2x}\ dx$

$du = dx \quad v = \dfrac{1}{2}e^{2x}$

$\displaystyle\int xe^{2x}\ dx = \frac{x}{2}e^{2x} - \int \frac{1}{2}e^{2x}\ dx$

$= \dfrac{x}{2}e^{2x} - \dfrac{1}{4}e^{2x} + C$

(b) $\displaystyle\int \frac{8}{x^2\sqrt{16-x^2}}\ dx, \qquad 0 < x < 4$

Solution:

$x = 4\sin\theta \quad -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$

$dx = 4\cos\theta\ d\theta$

$\sqrt{16-x^2} = \sqrt{16(1-\sin^2\theta)} = \sqrt{16\cos^2\theta} = 4|\cos\theta| = 4\cos\theta$

$\displaystyle\int \frac{8}{x^2\sqrt{16-x^2}}\ dx = \int \frac{8}{(4\sin\theta)^2\cdot 4\cos\theta}4\cos\theta\ d\theta = \frac{1}{2}\int \csc^2\theta\ d\theta$

$= -\dfrac{1}{2}\cot\theta + C$

$= -\dfrac{1}{2}\dfrac{\sqrt{16-x^2}}{x} + C$

13. (12 points) Evaluate the improper integral $\int_0^\infty \frac{1}{1+4x^2}\,dx$ if it converges or explain why it diverges.

Solution:

$\int \frac{1}{1+4x^2}\,dx = \frac{1}{2}\arctan 2x + C$

$\int_0^\infty \frac{1}{1+4x^2}\,dx = \lim_{t\to\infty}\int_0^t \frac{1}{1+4x^2}\,dx$

$= \lim_{t\to\infty} \frac{1}{2}\arctan 2x\Big|_0^t$

$= \frac{1}{2}\lim_{t\to\infty}\big(\arctan 2t - \arctan 0\big)$

$= \frac{\pi}{4}$

The integral converges.

14. (14 points) Solve the initial value problem. $y' = e^{-y}(2x-4), \quad y(5) = 0$

Solution:

$e^y \; dy = (2x-4) \; dx$

$\displaystyle\int e^y \; dy = \int (2x-4) \; dx$

$e^y = x^2 - 4x + C$

Solve for C using initial condition.

$e^0 = 5^2 - 4(5) + C$ gives $C = -4$

$y = \ln\left|x^2 - 4x - 4\right|$

15. (16 points) A vertical right cylindrical tank has height 2 ft and radius 1 ft. It is full of soda weighing 63 lbs/ft^3. How much work does it take to pump all of the soda from a tank to an outlet which is at the level of the top of the tank.

Solution:

$W = \displaystyle\int_a^b \sigma s(y) A(y) \; dy$

$\sigma = 63$ lbs/ft^3

$s(y) = 2 - y$

$A(y) = \pi(1)^2$

$W = \displaystyle\int_0^2 63(2-y)\pi(1)^2 \; dy$

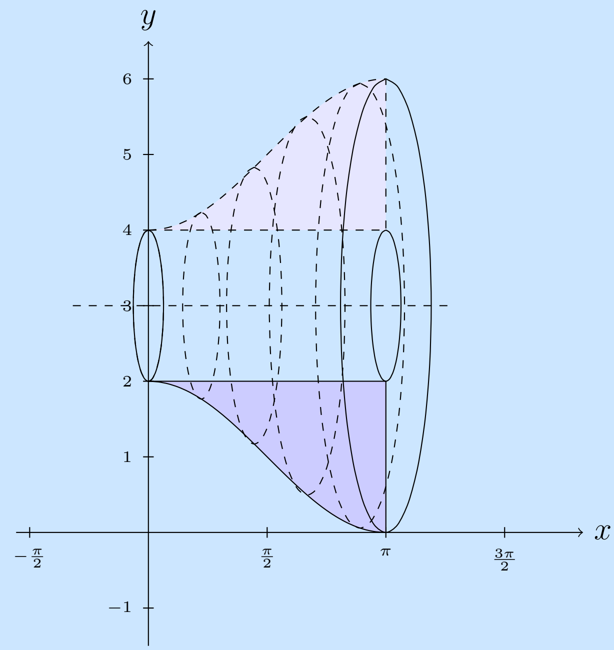
$= 63\pi \displaystyle\int_0^2 (2-y) \; dy$

$= 63\pi \left(2y - \dfrac{y^2}{2}\right)\Big|_0^2 = 126\pi$ ft-lbs

16. Find the volume of the solid generated by rotating the region of the xy-plane between $y = 2$, the curve $y = 1 + \cos(x)$, for $0 \le x \le \pi$ about the line $y = 3$.

(a) (4 points) Draw a picture of the solid.

Solution:



(b) (14 points) Set up and evaluate the integral representing the volume.

Solution:

$V = \int_0^{\pi} \left[\pi\big(3 - (1 + \cos x)\big)^2 - \pi(1)^2 \right] \, dx$

$= \pi \int_0^{\pi} \left[\big(2 - \cos x\big)^2 - 1 \right] \, dx$

$= \pi \int_0^{\pi} \big(3 - 4\cos x + \cos^2 x\big) \, dx$

Use power reducing formula $\cos^2 x = \dfrac{1}{2} + \dfrac{1}{2}\cos 2x$

$= \pi \int_0^{\pi} \left(\dfrac{7}{2} - 4\cos x + \dfrac{1}{2}\cos 2x \right) \, dx$

$= \pi \left(\dfrac{7}{2}x - 4\sin x + \dfrac{1}{4}\sin 2x \right) \Bigg|_0^{\pi}$

$= \dfrac{7}{2}\pi^2$ cubic units

Congratulations you are now done with the exam!

Go back and check your solutions for accuracy and clarity. Make sure your final answers are $\boxed{\text{BOXED}}$.

When you are completely happy with your work please bring your exam to the front to be handed in. **Please have your MSU student ID ready** so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	15	
3	20	
4	15	
5	16	
6	24	
7	12	
8	30	
9	18	
Total:	150	