Exercise 9.14

For lives \((x)\) and \((y)\) with independent future lifetimes, we have

\[
\text{Cov}[v_{T_{xy}}, v_{T_{xy}}] = E[v_{T_{xy}} \cdot v_{T_{xy}}] - E[v_{T_{xy}}] E[v_{T_{xy}}]
\]

\[
= E[v_{T_x} \cdot v_{T_y}] - E[v_{T_{xy}}] E[v_{T_{xy}}]
\]

\[
= E[v_{T_x}] E[v_{T_y}] - E[v_{T_{xy}}] E[v_{T_{xy}}]
\]

\[
= \bar{A}_x \bar{A}_y - \bar{A}_{xy} \bar{A}_{xy}
\]

\[
= \bar{A}_x \bar{A}_y - (\bar{A}_x + \bar{A}_x - \bar{A}_{xy}) \bar{A}_{xy}
\]

\[
= \bar{A}_x \bar{A}_y - \bar{A}_x \bar{A}_{xy} - \bar{A}_y \bar{A}_{xy} + (\bar{A}_{xy})^2
\]

\[
= (\bar{A}_x - \bar{A}_{xy})(\bar{A}_y - \bar{A}_{xy})
\]