Exercise 9.11

The APV of future benefits at issue is equal to

\[
\text{APV}(FB_0) = 100000 \bar{A}_{28:24} = 100000 \int_0^{\infty} v^t \, p^m_{28} \, p^f_{24} (\mu^m_{28+t} + \mu^f_{24+t}) \, dt,
\]

where the superscripts \(m\) and \(f\) refer to the husband and wife, respectively. Let \(G\) denote the annual gross premium so that the APV of future premiums is equal to

\[
\text{APV}(FP_0) = G \ddot{a}_{28:24:25}^{(12)}
\]

Finally, the APV of future expenses at issue is equal to

\[
\text{APV}(FE_0) = 250 - 0.03(G/12) + 0.03G \ddot{a}_{28:24:25}^{(12)}
\]

where here \(\ddot{a}_{28:24:25}^{(12)}\) denotes the APV of an annuity of $1 per year payable monthly so long as both (28) and (24) are alive, up to 25 years:

\[
\ddot{a}_{28:24:25}^{(12)} = \frac{1}{12} \sum_{k=0}^{25-(1/2)} v^{k/12} \, k/12 \, p^m_{28} \, k/12 \, p^f_{24}
\]

Solving for \(G\), we get

\[
G = \frac{100000 \bar{A}_{28:24} + 250}{0.97 \ddot{a}_{28:24:25}^{(12)} + (0.03/12)}
\]

\[
= \frac{100000(0.2484613) + 250}{0.97(13.32663) + (0.03/12)}
\]

\[
= 1941.024
\]

so that the monthly premium is

\[
G/12 = 1941.024/12 = 161.752.
\]

mu28m <- function(t){
A <- 0.0001
B <- 0.0004
c <- 1.075
A + B*c^(28+t)}
tp28m <- function(t){
A <- 0.0001
B <- 0.0004
c <- 1.075
temp <- A*t + B*c^(28*(c^t-1)/log(c)
exp(-temp))
mu24f <- function(t){
A <- 0.0001
B <- 0.0003
c <- 1.07
A + B*c^(24+t)}

\text{tp24f <- function(t){}
A <- 0.0001
B <- 0.0003
c <- 1.07
temp <- A*t + B*c^24*(c^t-1)/log(c)
exp(-temp)\}
i <- 0.05
v <- 1/(1+i)
h <- 1/1000
# limiting age
w <- 150
\text{tt <- max(w-28,w-24)
\text{t <- seq(0,tt,h)
\text{vt <- v^t}
\text{intA <- vt*tp28m(t)*tp24f(t)*(mu28m(t)+mu24f(t))
\text{apvfb <- 0
n <- 1
while (n<length(t)) {
\text{n <- n+2
} \}
k <- seq(0,25-(1/12),1/12)
\text{vk <- v^k}
\text{ann2824temp25 <- (1/12)*sum(vk*tp28m(k)*tp24f(k))
\text{num <- 100000*apvfb + 250
\text{den <- 0.97*ann2824temp25 + .03*(1/12)
\text{G <- num/den

> apvfb
[1] 0.2484613
> ann2824temp25
[1] 13.32663
> G
[1] 1941.024
> G/12
[1] 161.752

\text{Note the slight difference in the answer from that in the published text. This is because the}
\text{book included a 3\% of the first month’s premium for expenses, as indicated below:}

\[ G_{12} = \frac{100000 \cdot \bar{A}_{28:24} + 250}{0.97 \cdot \bar{a}_{28:24,25}^{[12]}} = \frac{100000(0.2484613) + 250}{0.97(13.32663)} = 161.7833 \]

\text{This is misleading because a renewal expense by definition does not include expense in the first}
\text{ premium.}