Exercise 8.4

(a) The reserve at time \( t \) for a policy in state 1 can be expressed as

\[
\frac{d}{dt} V^{(1)} = \delta t V^{(1)} - 50000 - \mu_{50+t}^{12} (200000 - t V^{(1)}) - \mu_{50+t}^{10} (t V^{(0)} - t V^{(1)})
\]

(b) The APV of future premiums is

\[
\text{APV(FP)} = P \bar{a}^{00}_{50} = 11.9520 P
\]

and the APV of future sickness benefits is

\[
\text{APV(FSB)} = 50000 \bar{a}^{01}_{50} = 50000 \times 1.3292
\]

and the APV of future death benefit is

\[
\text{APV(FDB)} = 200000 A^{02}_{50} = 200000 \times 0.34980.
\]

Solving for \( P \), we get

\[
P = 50000 \times \frac{1.3292 + 4 \cdot 0.34980}{11.9520} = 11,413.99.
\]

(c) The reserve at time 10 for the policy in state 1 is then

\[
V^{(1)}_{10} = \text{APV(FSB)}_{10} + \text{APV(FDB)}_{10} - \text{APV(FP)}_{10}
\]

\[
= 50000 \bar{a}^{11}_{60} + 200000 A^{12}_{60} - P \bar{a}^{10}_{60}
\]

\[
= 50000(7.1596) + 200000(0.56316) - 11413.99(1.7922)
\]

\[
= 450,155.80.
\]