Exercise 7.9

(a) Let $T_{[x]+t}$ denote the future lifetime of a life with select age $x$, $t$ years later. Therefore, the insurer’s loss at duration $t$ for this life can be expressed as

$$L^n_t = S v^{T_{[x]+t}} - P \bar{a}_{[x]+t} = \left( S + \frac{P}{\delta} \right) v^{T_{[x]+t}} - \frac{P}{\delta}.$$

The expectation of this loss at duration $t$ can be expressed as

$$E[L^n_t] = S \bar{A}_{[x]+t} - P \bar{a}_{[x]+t} = \left( S + \frac{P}{\delta} \right) \bar{A}_{[x]+t} - \frac{P}{\delta}$$

and the variance is

$$\text{Var}[L^n_t] = \left( S + \frac{P}{\delta} \right)^2 \text{Var}\left[v^{T_{[x]+t}}\right]$$

$$= \left( S + \frac{P}{\delta} \right)^2 \left[2 \bar{A}_{[x]+t} - (\bar{A}_{[x]+t})^2 \right].$$

(b) For $x = 55$ and $P = 1200$, by the equivalence principle, we have

$$S = P \times \frac{\bar{a}_{[55]}}{\bar{A}_{[55]}} = 1200 \times \frac{15.56159}{0.2407473} = 77566.44.$$

Here, as is the case with the rest of the problem, $\bar{a}_{[x]}$ is estimated using repeated Simpson’s rule with $h = 1/100$.

(c) One can verify the following calculations of the variances, and corresponding standard deviations, of the losses at durations 0, 5, and 10:

$$\text{Var}[L^n_0] = \left( S + \frac{P}{\delta} \right)^2 \left[2 \bar{A}_{[55]} - (\bar{A}_{[55]})^2 \right]$$

$$= \left( 77566.44 + \frac{1200}{\log(1.05)} \right)^2 \left[0.07821618 - (0.2407473)^2 \right] = 211421004$$

so that the standard deviation is

$$\text{SD}[L^n_0] = \sqrt{211421004} = 14540.32.$$

$$\text{Var}[L^n_5] = \left( S + \frac{P}{\delta} \right)^2 \left[2 \bar{A}_{[55]+5} - (\bar{A}_{[55]+5})^2 \right]$$

$$= \left( 77566.44 + \frac{1200}{\log(1.05)} \right)^2 \left[0.1137389 - (0.2974343)^2 \right] = 263760954$$

so that the standard deviation is

$$\text{SD}[L^n_5] = \sqrt{263760954} = 16240.72.$$
\[
\text{Var}[L^n_{10}] = \left( S + \frac{P}{\delta} \right)^2 \left[ 2\bar{A}_{[55]+10} - (\bar{A}_{[55]}+10)^2 \right] = \left( 77566.44 + \frac{1200}{\log(1.05)} \right)^2 [0.1618931 - (0.3635198)^2] = 310463852
\]
so that the standard deviation is
\[
\text{SD}[L^n_{10}] = \sqrt{310463852} = 17619.98.
\]
Thus, we observe an increasing variation in the loss with duration, partially due to the increasing variation of the timing of when the death payment is to be made.