Exercise 7.5

(a) Let $P$ denote the annual premium rate payable continuously. The present value of future benefits at issue can be expressed as

$$PV_{FB0} = \begin{cases} T_{30}Pv^{T_{30}}, & \text{for } T_{30} \leq 10 \\ 10Pv^{T_{30}}, & \text{for } 10 < T_{30} \leq 30 \\ v^{T_{30}}\ddot{a}(12)K_{30}^{(12)}+(1/12)−30, & \text{for } T_{30} > 30 \end{cases}$$

The present value of future premiums at issue can be expressed as

$$PV_{FP0} = \begin{cases} P\bar{a}_{T_{30}} & \text{for } T_{30} \leq 10 \\ P\bar{a}_{10} & \text{for } T_{30} > 10 \end{cases}$$

The future loss random variable $L_0$ is the difference between the two:

$$L_0 = PV_{FB0} - PV_{FP0} = \begin{cases} T_{30}Pv^{T_{30}} - P\bar{a}_{T_{30}} & \text{for } T_{30} \leq 10 \\ 10Pv^{T_{30}} - P\bar{a}_{10} & \text{for } 10 < T_{30} \leq 30 \\ v^{T_{30}}\ddot{a}(12)K_{30}^{(12)}+(1/12)−30 - P\bar{a}_{10}, & \text{for } T_{30} > 30 \end{cases}$$

(b) The actuarial present value of future premiums at issue is

$$APV(FP) = P\bar{a}_{30:10}$$

The actuarial present value of future benefits at issue is

$$APV(FB) = P(\bar{I}\bar{A})^{1}_{30:10} + 10P_{10}E_{30}\bar{A}^{1}_{40:20} + 30E_{30}\ddot{a}(12)\bar{a}_{60}.\$$

Equating the two APV’s by the equivalence principle, we get

$$P = \frac{30E_{30}\ddot{a}(12)\bar{a}_{60}}{\ddot{a}_{30:10} - (\bar{I}\bar{A})^{1}_{30:10} - 10_{10}E_{30}\bar{A}^{1}_{40:20}}.\$$

(c) For a policy still in force at duration 5, the present value of future loss random variable can be expressed as

$$L_5 = PV_{FB5} - PV_{FP5} = \begin{cases} (5 + T_{35})Pv^{T_{35}} - P\bar{a}_{T_{35}} & \text{for } T_{35} \leq 5 \\ 10Pv^{T_{35}} - P\bar{a}_{5} & \text{for } 5 < T_{35} \leq 25 \\ v^{25}\ddot{a}(12)K_{35}^{(12)}+(1/12)−25 - P\bar{a}_{5}, & \text{for } T_{35} > 25 \end{cases}$$

(d) For a policy still in force at duration 5, the policy value can be derived using

$$APV(FP_5) = P\bar{a}_{35:5}\$$

and

$$APV(FB_5) = 5\bar{A}^{1}_{35:5} + P(\bar{I}\bar{A})^{1}_{35:5} + 10P_{5}E_{35}\bar{A}^{1}_{40:20} + 25E_{35}\ddot{a}(12)\bar{a}_{60}.\$$

The policy value at duration 5 is therefore

$$5V = 5\bar{A}^{1}_{35:5} + P(\bar{I}\bar{A})^{1}_{35:5} + 10P_{5}E_{35}\bar{A}^{1}_{40:20} + 25E_{35}\ddot{a}(12) - P\bar{a}_{35:5}.\$$