Exercise 7.20

(a) Calculating the first year FPT premium, we have

\[ 1 P_{[50]} = 100,000 \times vq_{[50]} = 100,000 \times \left(1/1.04\right) \times \left[1 - (98450.67/98552.51)\right] = 99.36132 \]

If we let \( \beta \) to be the renewal premium, then it is clear that

\[ \beta = \frac{P \mathring{a}_{[50]:20} - 1 P_{[50]}}{\mathring{a}_{[50]:20} - 1} \]

where \( P \) is the net annual premium equal to

\[ P = 100,000 \frac{A_{[50]}}{\mathring{a}_{[50]:20}} = 100,000 \frac{0.255698}{13.86135} = 1844.684. \]

Plugging the appropriate values, we get

\[ \beta = \frac{1844.684(13.86135) - 99.36132}{13.86135 - 1} = 1980.386. \]

(b) First, consider gross premium valuation. At issue, the APV of future gross premiums is

\[ \text{APV}(FG) = G \mathring{a}_{[50]:20} \]

and the APV of future benefits is

\[ \text{APV}(FB) = 100000 \times A_{[50]} \]

and the APV of future expenses is

\[ \text{APV}(FE) = 0.47G + 225 + 0.03G \mathring{a}_{[50]:20} + 25 \mathring{a}_{[50]:20} \]

Thus, from equivalence principle, we have

\[ G = \frac{100000 \times A_{[50]} + 225 + 25 \mathring{a}_{[50]:20}}{0.9725 \mathring{a}_{[50]:20} - 0.47} = \frac{26141.33}{12.97551} = 2014.668. \]

Thus, the gross premium reserves for \( t = 0, 1, 2 \) and 10 are:

\[ _0V^g = 0 \]

\[ _1V^g = \frac{(_0V^g + 0.5G - 250)(1.04) - 100000q_{[50]}}{1 - q_{[50]}} = 684.9992 \]

where \( q_{[50]} = 0.00103358 \)

\[ _2V^g = \frac{(_1V^g + 0.97G - 25)(1.04) - 100000q_{[50] + 1}}{1 - q_{[50] + 1}} = 2595.639 \]
where \( q_{50} + 1 = 0.00126439 \)

Finally for \( t = 10 \), we have

\[
\begin{align*}
10 V^g &= 100000A_{60} + (25 - 0.97G)\ddot{a}_{60:10} \\
&= 100000(0.3629975) + (25 - 0.97(2014.668))(8.273434) \\
&= 20338.41
\end{align*}
\]

For net premium valuation, we have the net annual premium equal to \( P = 1844.684 \) from part (a). Thus, the net premium reserves for \( t = 0, 1, 2 \) and 10 are:

\[
\begin{align*}
0 V^n &= 0 \\
1 V^n &= \frac{(0 + P)(1.04) - 100000q_{50}}{1 - q_{50}} = 1817.013 \\
2 V^n &= \frac{(1 V^n + P)(1.04) - 100000q_{50} + 1}{1 - q_{50} + 1} = 3686.386 \\
10 V^n &= 100000A_{60} - P\ddot{a}_{60:10} = 100000(0.3629975) - (1844.684)(8.273434) = 21037.88
\end{align*}
\]

For FPT reserve calculation, we need the first and renewal year’s premiums computed in (a):

\[
\alpha = 99.36132 \quad \beta = 1980.386
\]

Thus, the FPT reserves for \( t = 0, 1, 2 \) and 10 are:

\[
\begin{align*}
0 V^{FPT} &= 0 \\
1 V^{FPT} &= 0 \\
2 V^{FPT} &= \frac{(1 V^{FPT} + \beta)(1.04) - 100000q_{50} + 1}{1 - q_{50} + 1} = 1935.61 \\
10 V^{FPT} &= 100000A_{60} - \beta\ddot{a}_{60:10} = 100000(0.3629975) - (1980.386)(8.273434) = 19915.15
\end{align*}
\]