Exercise 7.13

(a) Let $P$ be the annual benefit premium. The actuarial present value of future premiums at issue is

$$\text{APV} (FP_0) = P \dot{a}_{[40]}.$$ 

The actuarial present value of future benefits at issue is

$$\text{APV} (FB_0) = 1000 A_{[40]} + 49000 E_{[40]} A_{43}.$$ 

Using the actuarial equivalence principle and substituting values, we have

$$P = \frac{1000 A_{[40]} + 49000 E_{[40]} A_{43}}{\dot{a}_{[40]}} = \frac{1000(0.08424182) + 49000(0.8382802)(0.09880822)}{16.17839} = 256.0737.$$ 

(b) For $t \geq 3$, the policy value can be expressed as

$$tV = 50000 A_{[40]+t} - P \dot{a}_{[40]+t}.$$ 

(c) The policy value at $t = 3$ is

$$3V = 50000 A_{43} - P \dot{a}_{43} = 50000(0.09880822) - 256.0737(15.92105) = 863.4472.$$ 

(d) Starting with $0V = 0$, applying recursion relation, we have the policy values for the first two years as:

$$1V = \frac{(0V + P)(1 + i) - 1000 q_{[40]}}{1 - q_{[40]}} = \frac{256.0737(1.06) - 1000(0.0004506435)}{1 - 0.0004506435} = 271.1097$$

and

$$2V = \frac{(1V + P)(1 + i) - 1000 q_{[40]+1}}{1 - q_{[40]+1}} = \frac{(271.1097 + 256.0737)(1.06) - 1000(0.0005368943)}{1 - 0.0005368943} = 558.5774.$$ 

(e) The expected asset share per surviving policyholder at the end of year 3 is

$$\text{EA}_3 = \frac{(2V + P)(1 + 0.06) - 1000 q_{42}}{1 - q_{42}} = \frac{256.0737(1.06) - 1000(0.0004506435)}{1 - 0.0004506435} = 863.4472.$$
while the actual asset share per surviving policyholder is

$$AA_3 = \frac{(2V + P)(1 + 0.055) - 1000(4/985)}{1 - (4/985)}$$

$$= \frac{256.0737(1.06) - 1000(4/985)}{1 - (4/985)} = 858.8839.$$ 

Thus, the total profit for the year is the difference between the two multiplied by the number of actual surviving policyholders:

$$981 \times (858.8839 - 863.4472) = -4476.573.$$