Exercise 7.11

(a) At issue, the APV of future premiums is
\[ \text{APV}(FP_0) = G\bar{a}_{[50]}, \]
the APV of future benefits is
\[ \text{APV}(FB_0) = 10000A_{[50]}, \]
and the APV of future expenses is
\[ \text{APV}(FE_0) = 0.17G + 0.05G\bar{a}_{[50]} + 90 + 10\bar{a}_{[50]}. \]
By the equivalence principle, we have
\[ \text{APV}(FP_0) = \text{APV}(FB_0) + \text{APV}(FE_0) \]
and the gross annual premium, \( G \), is therefore
\[ G = \frac{10000A_{[50]} + 90 + 10\bar{a}_{[50]}}{0.95\bar{a}_{[50]} - 0.17}. \]

Based on the Standard Select Survival Model with \( i = 4.5\% \), one can verify that
\[ \bar{a}_{[50]} = 18.12588 \]
and that
\[ A_{[50]} = 1 - (0.045/1.045)(18.12588) = 0.2194596 \]
so that we have
\[ G = \frac{10000(0.2194596) + 90 + 10(18.12588)}{0.95(18.12588) - 0.17} = 144.6284. \]

(b) The policy value at each year end can be evaluated recursively. Starting with \( 0V = 0 \), we have
\[ 1V = \frac{(0V + 0.78G - 100)(1.045) - 10000q_{[50]}}{1 - q_{[50]}} = 3.056835 \]
where \( q_{[50]} = 0.001033293 \) based on the Standard Select Model. For \( k \geq 2 \), we have
\[ kV = \frac{(k-1V + 0.95G - 10)(1.045) - 10000q_{[50]+k-1}}{1 - q_{[50]+k-1}}. \]
The policy values, applying the recursion above, for the first few years are summarized in the table below:
Suggested solutions to DHW textbook exercises

(c) Since the only source of profit is from the higher than assumed interest rate in each year, we can compute the bonus at the end of year \( k \), denoted by \( \text{bonus}_k \), recursively as follows:

\[
\text{bonus}_1 = 0.90 \times \frac{(0V + 0.78G - 100)(0.055 - 0.045)}{1 - q_{[50]}} = 0.1154106
\]

and for \( k \geq 2 \), we have

\[
\text{bonus}_k = 0.90 \times \frac{(k-1)V + 0.95G - 10)(0.055 - 0.045)}{1 - q_{[50]+k-1}}.
\]

These values for the first few years are summarized below:

<table>
<thead>
<tr>
<th>( k )</th>
<th>( \text{bonus}_k )</th>
<th>( \text{bonus}_k )</th>
<th>( \text{bonus}_k )</th>
<th>( \text{bonus}_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.115</td>
<td>11 12.364</td>
<td>21 28.695</td>
<td>28.695</td>
</tr>
<tr>
<td>2</td>
<td>1.176</td>
<td>12 13.813</td>
<td>22 30.542</td>
<td>30.542</td>
</tr>
<tr>
<td>3</td>
<td>2.264</td>
<td>13 15.303</td>
<td>23 32.425</td>
<td>32.425</td>
</tr>
<tr>
<td>4</td>
<td>3.386</td>
<td>14 16.836</td>
<td>24 34.341</td>
<td>34.341</td>
</tr>
<tr>
<td>5</td>
<td>4.547</td>
<td>15 18.409</td>
<td>25 36.290</td>
<td>36.290</td>
</tr>
<tr>
<td>6</td>
<td>5.747</td>
<td>16 20.024</td>
<td>26 38.271</td>
<td>38.271</td>
</tr>
<tr>
<td>7</td>
<td>6.988</td>
<td>17 21.680</td>
<td>27 40.282</td>
<td>40.282</td>
</tr>
<tr>
<td>8</td>
<td>8.270</td>
<td>18 23.376</td>
<td>28 42.323</td>
<td>42.323</td>
</tr>
<tr>
<td>9</td>
<td>9.593</td>
<td>19 25.111</td>
<td>29 44.392</td>
<td>44.392</td>
</tr>
<tr>
<td>10</td>
<td>10.958</td>
<td>20 26.884</td>
<td>30 46.490</td>
<td>46.490</td>
</tr>
</tbody>
</table>

(d) The profit in year \( k \), denoted by \( \text{profit}_k \), can be recursively computed based on

\[
\text{profit}_1 = \frac{(0V + 0.78G - 100)(0.055 - 0.045)}{1 - q_{[50]}} = 0.1154106
\]

and for \( k \geq 2 \), we have

\[
\text{profit}_k = \frac{(k-1)V + 0.95G - 10)(0.055 - 0.045)}{1 - q_{[50]+k-1}}.
\]

These profit values are prior to the distribution of the 90% dividends to the policyholders. While it is not clear from the problem whether the expected present value required is for
profits prior to or after dividends, we give both values. For profits to the insurer prior to the distribution of dividends, the expected present value is given by

\[ \text{EPV(profits)} = \sum_k v^k \times k_{-1}p_{[50]} \times \text{profit}_k = 301.7388. \]

The expected present value is given by

\[ 0.10 \times 301.7388 = 30.17388 \]

after the 90% dividends distributed. Neither of this matches the given solution in the book because it is based on the following formula:

\[ \sum_k v^k \times k_{-1}p_{[50]} \times q_{[50]+k} \times \text{bonus}_k = 263.3683. \]

This computation clearly does not make sense in two respect: (i) it considers the bonus (and not the profit or remaining profit after bonus), and (ii) it does not consider value on a per policy basis.

(e) It is too early to be able to recoup a large proportion of the initial expense, for surrenders at the end of the first year. Hence, a reasonable surrender benefit is nothing.