Exercise 7.1

(a) Let $P$ be the required annual benefit premium and by the equivalence principle, we have

$$P = 200000 \times \frac{A^{\frac{1}{41}}_{[3]} - 3E_{41}}{\ddot{a}^{\frac{1}{41}}_{[3]}},$$

where

$$\ddot{a}^{\frac{1}{41}}_{[3]} = 1 + vp_{41} + v^2p_{41} = 1 + \frac{1}{1.06} \cdot 99689 + \frac{1}{1.06^2} \cdot 99502 = 2.829644$$

and

$$A^{\frac{1}{41}}_{[3]} = A_{[3]} - 3E_{41} = (1 - \ddot{a}^{\frac{1}{41}}_{[3]}) - v^3p_{41}$$

$$= \left[1 - (1 - (1.06)^{-1})(2.829644)\right] - \frac{1}{1.06^3} \cdot 99283 = 0.004578162.$$

Thus, it follows that

$$P = 200000 \times \frac{0.004578162}{2.829644} = 323.5851.$$

(b) Simply denote $K_{41+1}$ by $K$. We have

$$L_1 = PVFB_1 - PVFP_1 = \begin{cases} 200000v^{K+1} - P\ddot{a}^{K+1}_{[3]} & \text{for } K < 2 \\ -P\ddot{a}^{K+1}_{[3]} & \text{for } K \geq 2 \end{cases}$$

The following table provides details of the calculations for the expected value and standard deviation of $L_1$:

<table>
<thead>
<tr>
<th>$k$</th>
<th>$Pr[K = k]$</th>
<th>$L_1 = \ell \cdot Pr[K = k]$</th>
<th>$\ell^2 \cdot Pr[K = k]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.001875834</td>
<td>200000v - $P = 188355.6602$</td>
<td>353.3239</td>
</tr>
<tr>
<td>1</td>
<td>0.002196832</td>
<td>200000v^2 - $P(1 + v) = 177370.4339$</td>
<td>389.6531</td>
</tr>
<tr>
<td>$\geq$ 2</td>
<td>0.995927334</td>
<td>$-P(1 + v) = -628.8541$</td>
<td>-626.2930</td>
</tr>
<tr>
<td>sum</td>
<td>1.00000</td>
<td>116.6840</td>
<td>136057342</td>
</tr>
</tbody>
</table>

Thus, we find from this table that

$$E[L_1] = 116.6840 \text{ and } E[L_1^2] = 136057342$$

so that the required standard deviation is given by

$$SD[L_1] = \sqrt{E[L_1^2] - (E[L_1])^2} = \sqrt{136057342 - (116.6840)^2} = 11663.78.$$
(c) Let $B$ be the required sum insured so that $B$ satisfies

$$P \times \ddot{a}_{[41]:3} = B \times A_{[41]:3}$$

The solution is therefore

$$B = P \times \frac{\ddot{a}_{[41]:3}}{A_{[41]:3}} = P \times \frac{\ddot{a}_{[41]:3}}{1 - (1 - v)\ddot{a}_{[41]:3}}$$

$$= 323.5851 \times \frac{2.829644}{1 - (1 - (1/1.06))2.829644} = 1090.258.$$ 

(d) Following the procedure in (b), we provide the table below for the details of the calculations for the expected value and standard deviation of the corresponding $L_1$:

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\Pr[K = k]$</th>
<th>$L_1 = \ell \cdot \Pr[K = k]$</th>
<th>$\ell^2 \cdot \Pr[K = k]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.001875834</td>
<td>$Bv - P = 704.9599$</td>
<td>1.322388</td>
</tr>
<tr>
<td>$\geq 1$</td>
<td>0.998124166</td>
<td>$Bv^2 - P(1 + v) = 341.4714$</td>
<td>340.830815</td>
</tr>
<tr>
<td>sum</td>
<td>1.00000</td>
<td>342.1532</td>
<td>117316.2</td>
</tr>
</tbody>
</table>

Thus, we find from this table that

$$E[L_1] = 342.1532 \text{ and } E[L_1^2] = 117316.2$$

so that the required standard deviation is given by

$$SD[L_1] = \sqrt{E[L_1^2] - (E[L_1])^2} = \sqrt{117316.2 - (342.1532)^2} = 15.72824.$$ 

(e) Because of the extra payment of the pure endowment in an endowment policy, this leads to a larger expected future loss, but smaller variation than that of a term insurance.