Exercise 6.20

(a) Let $P$ be the required annual premium. By the equivalence principle, we have

$$P = 250000 \frac{A_{[40]:[20]}}{\ddot{a}_{[40]:[20]}}$$

where

$$\ddot{a}_{[40]:[20]} = \ddot{a}_{[40]} - 20E_{[40]}\ddot{a}_{60} = \ddot{a}_{[40]} - v^{20} \frac{\ell_{60}}{\ell_{[40]}} \ddot{a}_{60}$$

$$= 18.45956 - (1.05)^{-20} \frac{96634.14}{99327.82} \cdot 14.90407 = 12.99471$$

and

$$A_{[40]:[20]} = 1 - (1 - (1/1.05))(12.99471) = 0.3812045,$$

giving us

$$P = 250000 \frac{0.3812045}{12.99471} = 7333.842.$$  

(b) Based on the equivalence principle, $E[L_0] = 0$ and since, in this case, we have

$$L_0 = \left(250000 + \frac{P}{d}\right) v^{\min(K+1, 20)} - \frac{P}{d},$$

then

$$\text{Var}[L_0] = \left(250000 + \frac{P}{d}\right)^2 \text{Var}[v^{\min(K+1, 20)}] = \left(250000 + \frac{P}{d}\right)^2 \left[2A_{[40]:[20]} - (A_{[40]:[20]})^2\right],$$

where

$$2A_{[40]:[20]} = 2A_{[40]} - v^{40} \frac{\ell_{60}}{\ell_{[40]}} A_{60} + v^{40} \frac{\ell_{60}}{\ell_{[40]}}$$

$$= 0.02338 - (1.05)^{-40} \frac{96634.14}{99327.82} (0.10834) + (1.05)^{-40} \frac{96634.14}{99327.82}$$

$$= 0.1466016$$

so that the standard deviation of $L_0$ is

$$\text{SD}[L_0] = \left(250000 + \frac{7333.842}{1 - (1/1.05)}\right) \sqrt{0.1466016 - (0.3812045)^2}$$

$$= 14481.31.$$  

The answer is off a bit from the textbook answer.

(c) For 10,000 identical, independent contracts, the 99th percentile of the (aggregate) net future loss is given by

$$z_{0.99} \times \sqrt{10000} \times \text{SD}[L_0] = 2.326(100)(14481.31) = 3368353,$$

based on the Normal approximation with $z_{0.99}$ denoting the 99th percentile of a standard Normal. The answer does not match the textbook answer.