Exercise 6.18

With the extra risk, we have

\[ p'_x = \exp \left[ - \int_0^t \mu'_{x+s} ds \right] = \exp \left[ - \int_0^t (\mu_{x+s} + \phi) ds \right] = \exp \left[ - \int_0^t \mu_{x+s} ds \right] \cdot e^{-\phi t} = p_x \cdot e^{-\phi t} \]

so that we have

\[ \bar{A}'_x = \int_0^\infty v^t p'_x \mu'_{x+t} dt \]
\[ = \int_0^\infty v^t p_x \cdot e^{-\phi t} (\mu_{x+t} + \phi) dt \]
\[ = \int_0^\infty e^{-(\delta+\phi)t} p_x \mu_{x+t} dt + \phi \int_0^\infty e^{-(\delta+\phi)t} p_x dt \]
\[ = \bar{A}'_x + \phi \bar{a}'_x \]

where \( \bar{A}'_x \) and \( \phi \bar{a}'_x \) are respectively, continuous whole life insurance and whole life annuity evaluated at the force of interest

\[ \delta' = \delta + \phi. \]

Equivalently, this leads to an annual effective interest rate of

\[ j = (1 + i) e^\phi - 1. \]