Exercise 6.17

(a) \( P = 30000a_{[60]} = 30000(\bar{a}_{[60]} - 1) = 30000(14.913 - 1) = 417,390 \)

(b) We can indeed write the present value of profit as

\[
\text{Profit} = -L_0 = P - 30000a_{\bar{K}_1}
\]

The event profit is positive is equivalent to the event

\( P > 30000a_{\bar{K}_1} \)

This event is equivalent to

\[
K < -\frac{1}{\delta} \log \left( 1 - \frac{Pi}{30000} \right) = -\frac{1}{\log(1.05)} \log \left( 1 - \frac{(417390)(0.05)}{30000} \right) = 24.38149.
\]

Thus, we have

\[
\Pr[\text{Profit} > 0] = \Pr[K < 24.38149] = \Pr[K \leq 24]
\]

\[
= 1 - 25p_{[60]} = 1 - \frac{\ell_{85}}{\ell_{[60]}} = \frac{61184.88}{96568.13} = 0.3664071.
\]

(c) The variance of the present value of profit is

\[
\text{Var}[\text{Profit}] = \text{Var}[L_0] = \left( \frac{30000}{d} \right)^2 \left[ 2A_{[60]} - (A_{[60]})^2 \right]
\]

\[
= \left( \frac{30000}{0.05/1.05} \right)^2 \left[ 0.10781 - (0.28984)^2 \right] = 9447321159,
\]

so that the standard deviation is

\[
\text{SD}[\text{Profit}] = 97197.33.
\]

(d) For a 1000 such annuities, the total profit is

\[
\text{TP} = -\sum_{i=0}^{1000} L_{0,1} = -\sum_{i=0}^{1000} \left( P - 30000a_{\bar{K}_1} \right)
\]

where its mean is

\[
\text{E}[\text{TP}] = -1000(P - 417390)
\]

and its standard deviation is

\[
\text{SD}[\text{TP}] = \sqrt{1000} \times 97197.33.
\]
Thus we require

\[ \Pr[TP > 0] = 0.95 \iff \Pr \left[ Z > \frac{1000(P - 417390)}{\sqrt{1000(97197.33)}} \right] = 0.95 \]

Solving for \( P \) from

\[ \frac{1000}{\sqrt{1000}} \frac{P - 417390}{97197.33} = 1.645, \]

we get

\[ P = 417390 + 1.645 \times \frac{\sqrt{1000}}{1000} 97197.33 = 422446.2. \]