STT 456 Review Problems for Class Test 1  
February 25, 2015

1. An insurance company sells 1,000 fully discrete whole life insurance contracts of $1, each to the same age 50. You are given:

- All contracts have independent future lifetimes.
- There are no expenses.
- Mortality follows the Standard Ultimate Survival Model with $i = 5\%$.

Using the Normal approximation, calculate the annual contract premium, for each policy, according to the portfolio percentile premium principle so that the company has at least a 95% probability of a positive gain from this portfolio of contracts.

2. For a special whole life insurance on (45), you are given:

- Benefit is paid at the end of the year of death. The death benefit is $100,000 for the first 20 years and reduces to $50,000 thereafter.
- The annual benefit premium of $4,945 is payable once at the beginning of each year for the first 20 years only; no premiums are payable after 20 years.
- The following actuarial present values:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$A_x$</th>
<th>$\ddot{a}_x$</th>
<th>$10E_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>0.5628</td>
<td>4.8091</td>
<td>0.0758</td>
</tr>
<tr>
<td>65</td>
<td>0.7532</td>
<td>2.7147</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

Calculate the benefit reserve at the end of 10 years.

3. For a fully discrete whole life insurance of $1,000 on (x), you are given:

- The expense, incurred at the beginning of each year, is 10% of the annual benefit premium.
- The gross premium reserve at the end of policy year $k$ is 602.45.
- The gross premium reserve at the end of policy year $k + 1$ is 629.72.
- $A_x = 0.6135$
- $i = 5\%$

Calculate $q_{x+k}$.

4. An insurer issued 400,000 fully discrete whole life insurance policies to lives all exactly age 50 on January 1, 2002. Each policy issued has a death benefit of $100,000 with an annual gross premium of $2,600.

You are given:
• The following values in Year 2011:

<table>
<thead>
<tr>
<th>Expenses as a percent of premium</th>
<th>anticipated</th>
<th>actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual effective rate of interest</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>( q_{59} )</td>
<td>0.0085</td>
<td>0.0090</td>
</tr>
</tbody>
</table>

• The gross premium reserves per policy at the end of Year 2010 and Year 2011, respectively, are:

\[ 9V = 2,044.32 \text{ and } 10V = 2,324.13 \]

• A total of 385,100 remain in force at the beginning of Year 2011.

• Gains and losses are calculated in the following order: interest then expenses then mortality.

Calculate the total gain (or loss) due to interest for this portfolio of policies in Year 2011.

5. For a life insurance policy issued to \( (x) \), you are given:

• Death benefit of $1 is paid at the end of the year of death.
• The benefit premium in year 11, payable at the beginning of the year, is $0.045.
• There are no expenses for this policy.
• The policy is still active after 10 years.
• Deaths are assumed to be uniformly distributed over integral ages.
• \( 10V = 0.325 \)
• \( p_{x+10} = 0.925 \)
• \( i = 6\% \)

Calculate \( 10.4V \).

6. The joint lifetime of a husband \( (x) \) and a wife \( (y) \) is being modeled as:

\[
\begin{align*}
(x) \text{ alive} & \quad (y) \text{ alive} & \quad (x) \text{ alive} \\
0 & \quad \mu^f_{y+t} & \quad 1 \\
\mu^m_{x+t} & \quad & \mu^m_{x+t} \\
(x) \text{ dead} & \quad (y) \text{ alive} & \quad (x) \text{ dead} \\
2 & \quad \mu^f_{y+t} & \quad 3 \\
\end{align*}
\]
You are given:

\[ \mu_{x+t}^m = 0.03, \text{ for all } t > 0 \]
and

\[ \mu_{y+t}^f = 0.02, \text{ for all } t > 0 \]

Calculate the probability that \((x)\) and \((y)\), given both are alive today, will be dead within the next 10 years.