Exercise 6.9

Let $G$ the required gross monthly premium.

The APV of the 20-year deferred annuity benefits with an initial annual payment of 50,000 increasing by 2% thereafter is given by

$$\text{APV(benefits)} = \sum_{k=20}^{\infty} 50000(1.02)^{k-20} v^k kP_{[40]}$$

$$= \frac{50000}{1.02^{20}} \left[ \sum_{k=20}^{\infty} (1.02v)^k kP_{[40]} \right]$$

$$= \frac{50000}{1.02^{20}} \left( \frac{\ddot{a}_{40}}{20} \right)_{i_1},$$

where $\left( \ddot{a}_{40} \right)_{i_1}$ is a 20-year deferred annuity evaluated at interest rate $i_1 = (1.05/1.02) - 1$. It can be verified that based on the Standard Select Survival Model, we have

$$\left( \ddot{a}_{40} \right)_{i_1} = 10.18434.$$

The APV of the expenses can be found using

$$\text{APV(expenses)} = 0.025(50000) + 0.15G + 0.05(12G) \ddot{a}_{[40]:20}^{(12)} + \sum_{k=0}^{\infty} 20(1.03)^{k+1} v^{k+1} kP_{[40]} q_{[40]+k}$$

$$= 1250 + 0.15G + 0.60G \ddot{a}_{[40]:20}^{(12)} + 20(A_{[40]})_{i_2},$$

where $(A_{[40]})_{i_2}$ is a whole life insurance of 1 with benefit payable at the end of the year of death, evaluated at interest rate $i_2 = (1.05/1.03) - 1$. It can be verified that based on the Standard Select Survival Model, we have

$$(A_{[40]})_{i_2} = 0.4245105.$$

The APV of the monthly gross premiums is given by

$$\text{APV(premiums)} = 12G \ddot{a}_{[40]:20}^{(12)},$$

where we can approximate the temporary annuity using the Woolhouse formula, with three terms:

$$\ddot{a}_{[40]:20}^{(12)} \approx \ddot{a}_{[40]:20} - \frac{11}{24} (1 - 20E_{[40]}) - \frac{12^2 - 1}{12(12^2)} \left[ \delta + \mu_{[40]} - 20E_{[40]} (\delta + \mu_{60}) \right],$$

where $\delta = \log(1.05)$ and

$$20E_{[40]} = v^{20} \frac{\ell_{60}}{\ell_{40}} = (1.05)^{-20} \frac{96634.14}{99327.82} = 0.3666686$$

$$\ddot{a}_{[40]:20} = \ddot{a}_{[40]} - 20E_{[40]} \ddot{a}_{60} = 18.45956 - 0.3666686(14.90407) = 12.99471$$

$$\mu_{[40]} = (0.9)^2(A + Bc^{40}) = 0.0004128936$$

$$\mu_{60} = A + Bc^{60} = 0.003221528$$

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Plug these values, we get

\[
\ddot{a}_{40:30}^{(12)} \approx 12.70194.
\]

Equating APV(benefits) + APV(expenses) with APV(premiums), we solve the monthly gross premium with

\[
G = \frac{(50000/1.02^{20})(\ddot{a}_{40:30})_{i_1} + 1250 + 20(A_{40})_{i_2}}{11.4 \ddot{a}_{40:30}^{(12)} - 0.15} - 12.70194 - 0.15
\]

= \frac{(50000/1.02^{20})(10.18434) + 1250 + 20(0.4245105)}{11.4(12.70194) - 0.15}

= 2377.754.