Exercise 6.7

(a) Let $P$ be the net single premium. The net future loss random variable can be written as

$$L_0 = P v^{K+1} I(K < 19) + 40000 v^{20} \ddot{a}_{K+1:20} - P$$

(b) Solving for $P$, we get

$$P = P A_{[45]:20}^1 + 40000 E_{[45]} \ddot{a}_{65}$$

so that

$$P = \frac{40000 E_{[45]} \ddot{a}_{65}}{1 - A_{[45]:20}^1}$$

$$= \frac{40000(0.35999)(13.550)}{1 - (0.15149 - 0.35999(0.35477))} = \frac{195114.6}{0.9762237} = 199,866.7$$

(c) With the 5 year guarantee, the actuarial present value of benefits can be expressed as

$$\text{APV(benefits)} = P A_{[45]:20}^1 + 40000 E_{[45]} \left( \ddot{a}_{51} + 5 E_{65} \ddot{a}_{70} \right)$$

so that

$$P = \frac{40000 E_{[45]} \left( \ddot{a}_{51} + 5 E_{65} \ddot{a}_{70} \right)}{1 - A_{[45]:20}^1}$$

$$= \frac{40000(0.35999) \left( \frac{1 - v^5}{d} + 0.75455(12.008) \right)}{1 - (0.15149 - 0.35999(0.35477))}$$

$$= \frac{195929.4}{0.9762237} = 200,701.4$$