Exercise 5.17

(a) For a whole life annuity-due on (65), we can write the present value random variable of the benefits as

\[ Y = \ddot{a}_{65+1} \]

where \( K \) is the curtate future lifetime of (65) with probability mass

\[ \Pr[K = k] = kp_{65}q_{65+k}. \]

The following R code calculates the expected value and variance of \( Y \) based on the Standard Ultimate Survival Model with \( i = 5\% \):

```r
# whole life annuity-due on (65)
A <- .00022
B <- 2.7*10^(-6)
c <- 1.124
surv <- function(x){
  exp(-A*x-(B*(c^x-1)/log(c)))
} x <- 65:137 px <- surv(x+1)/surv(x) qx <- 1-px int <- .05 v <- 1/(1+int) vcum <- v^(0:(length(x)-1)) y <- cumsum(vcum) prob <- cumprod(c(1,px[-length(px)])) qx) prob[length(prob)] <- 1 - sum(prob[-length(prob)]) EY <- sum(y*prob) EY2 <- sum(y^2 * prob) VarY <- EY2 - EY^2
```

This produces the results:

```r
> EY
[1] 13.54979
> VarY
[1] 12.49732
```

(b) With a 10-year annuity guarantee, we can write the present value random variable of the benefits as

\[ Y = \ddot{a}_{10}I(K \leq 9) + \ddot{a}_{K+11}I(K > 9). \]

The following R code calculates the expected value and variance of \( Y \) based on the Standard Ultimate Survival Model with \( i = 5\% \):
# whole life annuity-due on (65), with 10 year guarantee
# replace the first 10 years of benefit with a 10-year annuity-due, guaranteed

```r
y[1:10] <- y[10]
EY <- sum(y*prob)
EY2 <- sum(y^2 * prob)
VarY <- EY2 - EY^2
```

This produces the results:

```r
> EY
[1] 13.81410
> VarY
[1] 8.379656
```

A life annuity with a guarantee leads to some fixed, but more expensive and with less variable benefit payments. Hence, the life annuity with guarantee has a higher expectation but lower variance.