Exercise 4.5

(a) Endowment insurance is the sum of a term insurance and a pure endowment, that is, $A_{x:n} = A_{x:n|}^1 + A_{x:n}^1$. Therefore, we have

$$A_{x:n} = \sum_{k=0}^{n-1} v^{k+1} k q_x + v^n n p_x$$

$$= \sum_{k=0}^{n-2} v^{k+1} k q_x + v^n_{n-1} q_x + v^n n p_x$$

$$= \sum_{k=0}^{n-2} v^{k+1} k q_x + v^n \left[ n_{n-1} p_x (1 - p_{x+n-1}) + n p_x \right]$$

$$= \sum_{k=0}^{n-2} v^{k+1} k q_x + v^n \left[ n_{n-1} p_x - \frac{n_{n-1} p_x p_{x+n-1} + n p_x}{n p_x} \right]$$

$$= \sum_{k=0}^{n-2} v^{k+1} k q_x + v^n n_{n-1} p_x$$

which proves the result.

(b) The primary difference lies in the payment after period $n - 1$. If ($x$) survives to live for $n - 1$ years, he will receive a benefit at the end of year $n$ if he dies the following year, and if he survives, he will receive the pure endowment. So once he reaches age $x + n - 1$, a benefit of $1$ is payable at the end of year $n$, regardless of whether he survives or not. Hence, this explains the second term $v^n n_{n-1} p_x$ in the equation in part (a).