Exercise 4.11

Consider any $i^* > i$ and let $v = 1/(1 + i)$ and $v_* = 1/(1 + i^*)$. Then clearly $v > v_*$ so that

$$(\bar{A}_x)_i = \int_0^\infty v^t f_x(t) dt$$

$$> \int_0^\infty v_*^t f_x(t) dt = (\bar{A}_x)_{i^*}$$

The result immediately follows. Another way to prove this is to take the derivative of $\bar{A}_x$ with respect to $i$. First note that

$$\frac{d}{di} v^t = -t \cdot v^t \cdot \frac{d\delta}{di} = -t \cdot v^t \cdot v = -t \cdot v^{t+1}.$$ 

It follows therefore that

$$\frac{d}{dt} \bar{A}_x = \frac{d}{di} \int_0^\infty v^t f_x(t) dt = -\int_0^\infty t v^{t+1} f_x(t) dt < 0.$$

For larger interest rate, the time value of money leads to smaller present values and thus smaller actuarial present values.