Exercise 4.10

One way to approach this problem is to use random variables. Recall that we can write

$$T = K + S$$

where $S \sim U(0,1)$ and $K, S$ are independent in the case where UDD is assumed. Thus, we have

$$\bar{I} \bar{A}_x = E[Tv^T] = E[(K + S)v^{K+S}]$$

$$= E[(K + 1 + S - 1)v^{K+1+S-1}]$$

$$= E[(K + 1)v^{K+1}v^{S-1}] + E[v^{K+1}(S - 1)v^{S-1}]$$

$$= E[v^{S-1}]E[(K + 1)v^{K+1}] + E[(S - 1)v^{S-1}]E[v^{K+1}]$$

$$= E[v^{S-1}](IA)_x + E[(S - 1)v^{S-1}]A_x$$

When $S$ is uniformly distributed on $(0,1)$, it can be shown that

$$E[v^{S-1}] = \int_0^1 v^{s-1}ds = \frac{i}{\delta}$$

and that

$$E[(S - 1)v^{S-1}] = \int_0^1 (s - 1)v^{s-1}ds = \frac{i - \delta e^\delta}{\delta^2}$$

so that

$$\bar{I} \bar{A}_x = \frac{i}{\delta}(IA)_x + \frac{i - \delta e^\delta}{\delta^2}A_x$$