Exercise 2.16

Start with:

\[ e_{x\cdot n} = \mathbb{E} \left[ \min(K_x, n) \right] = \sum_{k=0}^{\infty} \min(k, n) q_x = \sum_{k=1}^{n-1} k_{k|q_x} + n \sum_{k=n}^{\infty} k_{q_x} \]

Now by noting that

\[ \sum_{k=1}^{n-1} k_{k|q_x} = \sum_{k=1}^{n-1} k \left( p_x - (k+1)p_x \right) = \sum_{k=1}^{n-1} kp_x - (n-1)p_x \]

and that

\[ \sum_{k=n}^{\infty} k_{q_x} = np_x, \]

we get the desired result:

\[ e_{x\cdot n} = \sum_{k=1}^{n-1} kp_x - (n-1)p_x + np_x = \sum_{k=1}^{n} kp_x \]